An original method of direct calculation for the identification of the last hinge and the definition of the deformative state at collapse

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ABSTRACT. The subject matter of these notes refers to the Ultimate Strength Design of 2-D steel framed structures and in particular to the analysis of the deformation state at collapse.

The idea is based on the consideration that, if the structure at its collapse condition is subjected to an articulated movement, similar and concordant to the crisis motion, this will not change the stress state of the system. This motion is known once a single parameter is fixed, namely the displacement of a point or the rotation of a beam.

When the collapse mechanism of the structure is already determined through any instrument of the Limit Analysis, a subsequent (k+1) plastic hinge can be arbitrarily fixed and assumed as the last developed one. It is therefore possible to solve the modified scheme through the rotation method and make a comparison in the verses between the known plastic moments and the rotations at the corresponding hinges. If the comparison is successful, in the sense that the checked verses are concordant, the selected hinge is actually the one formed as the last. On the contrary, the rotations resulting from an imprinted motion in the verse of collapse movement are algebraically added. If, for each hinge, the product between the plastic moment and the correspondent algebraic sum is made, this product has to be surely positive, due to the verses concordance. This can be translated in k+1 inequalities, with each one furnishing a lower limit for the parameter from which the articulated motion depends. Among these, the highest value is that one which makes all the inequalities to be simultaneously verified. The substitution of this value into the expressions for rotations permits to arrive to the simultaneous identifying of the last hinge and of the complete picture of deformations.

KEYWORDS. Final Hinge; Plastic Hinge; Plastic Moments; Collapse.

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INTRODUCTION

It is made reference to the previously wrote monograph titled "Un procedimento originale per l'individuazione della cerniera ultima e per la determinazione della deformazione completa allo stato di collasso", published in the journal LaborEst n.11 [1].
The topic is taken up in relation to the fact that, thanks to the introduction of a fundamental concept, it has been possible to arrive to the solution of the same problem through direct calculation and therefore not through an iterative procedure.

CONCISE CONTENTS OF THE ABOVE-MENTIONED MONOGRAPH

The main aspects that led to the solution of the problem can be subdivided in the following points:
a. demonstration that it is necessary to individualize the last plastic hinge with the aim to know the deformation state at collapse;
b. formulation, by means suitable analyses, of the Last Hinge Theorem, that can be stated as follows: given a framed structure, k-times hyper-static, whose collapse mechanism is known, if one of the k+1 plastic hinges is chosen as the last one, therefore the concordance of verses between plastic moments and rotations at all the k plastic hinges is necessary and sufficient condition to assert that this configuration is the real one;
c. choice of any of the k+1 plastic hinges and assumption that it is the last one to be formed, and solution of the structure through the rotation method. Knowing the deformed state of the system, this method allows to proceed to the comparison of verses of plastic moments and rotations of the correspondent hinges. If, according to what already said, the comparison between verses concordance returns a positive result, then the selected hinge is the last one to be formed. On the contrary, the evaluation must be repeated considering a different hinge as the last one. The procedure will end once the above concordance will be satisfied for all the k residual plastic hinges. In this case, the last plastic hinge will be known as well as the complete deformation of the system (rotations at all the plastic hinges and displacements).
The procedure illustrated above inevitably needs, for each iteration, the compilation and solution of a system of equations with the aim to obtain the deformative state of the structure in relation to the position of the plastic hinge assumed as the last one to be formed.

THE NEW PROCEDURE

In this work an innovative criterion is presented. It moves from the consideration that, if the structure at its collapse condition is subjected to an articulated movement, similar and concordant to the crisis motion, this will not change the stress state of the system because at this stage all the bending moments already reached their pick values. The imposed articulated movement produces the kinematical effect to modify the deformative state of the system, determining the passage from one configuration to another one. This motion is known once a single parameter is fixed, namely the displacement of a point or the rotation of a beam. It can be therefore inferred that a bijective correspondence exists between this parameter and the resulting deformed configuration: the one is known once the other is fixed and vice-versa.
On the basis of what above said and under the hypothesis that the kinematics at collapse is known by applying any suitable procedure (in particular the first or second theorem of the Limit Analysis [2, 3]), one of the k+1 plastic hinges can be arbitrarily fixed and, under the hypothesis that this is the last one to be formed, the obtained scheme can be studied through the rotation method [4]. The comparison, in verses, between the plastic moments (known) and rotations of the correspondent hinges (obtained by applying the above method) can be now carried out.
It is useful to denote the set of these deformations as "configuration 1".
If the comparison returns a positive result in terms of verses concordance, then the selected hinge is that one really formed as the last one and the search for it ends here. But the most interesting case (and that is clearly the most frequent) is when the comparison reveals at least one discordance.
In the second case, an articulated movement in the sense of the collapse motion is imposed through the choice of a unique parameter, so that a deformed configuration is obtained where each plastic hinge has a rotation whose value is dependent by this parameter.
It is useful to denote the set of these deformations as "configuration 2".

The rotations of the two above-mentioned configurations are now algebraically summed up. These are obviously functions of the parameter that originated the configuration 2. By making, for each plastic hinge, the product between the bending moment and the correspondent rotation (which is the sum of configurations 1 and 2), the result, for the verses concordance, must be necessarily negative. This leads to k+1 inequalities, with each one furnishing a lower limit for the parameter α from which the configuration 2 depends. Clearly, the highest value between these limits is that one which simultaneously verifies all the inequalities.
The substitution of this value in the expressions for rotations, permits to come to the simultaneous identification of the plastic hinge and of the complete picture of deformations.

CONVENTION ON SIGNS

As it is known, using the displacement method for applications to 2-D framed structures, the rotations at the extremes of a beam are assumed positive if clockwise. The sign convention, according to the Fracture Calculus, is different. There is therefore the problem to adequate one convention to the other one. To this end, if the beam is arranged in a horizontal way and with the stretched fibers pointed downwards (Fig. 1), the rotation at the right extreme of the beam is transferred to the plastic hinge with the same sign, the rotation to the left extreme of the beam is instead transferred with the opposite sign.

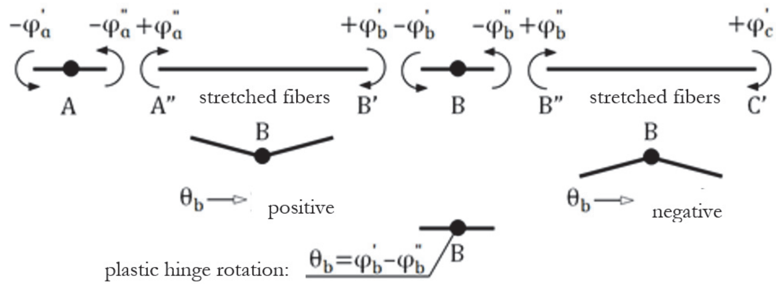

Figure 1: Sign convention.

The rotation of the plastic hinge will result, in value and sign, equal to the algebraic sum of the two rotations. As a consequence, if the angle formed by the two beams adjacent to the plastic hinge increases, the rotation of the plastic hinge will be positive; negative on the contrary (Fig. 1).

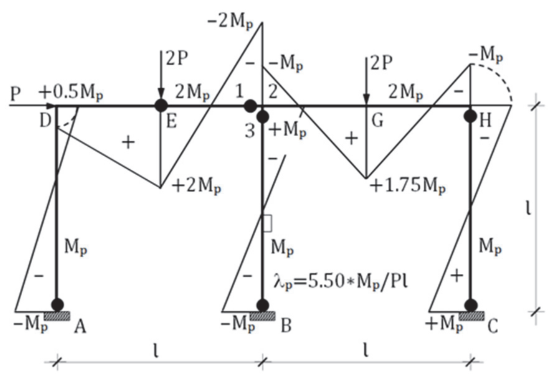

Figure 2: Collapse scheme.

NUMERICAL APPLICATION

With the aim to make the above theory more accessible, it is considered worthwhile to solve in detail the case of the frame reported in Fig. 2.

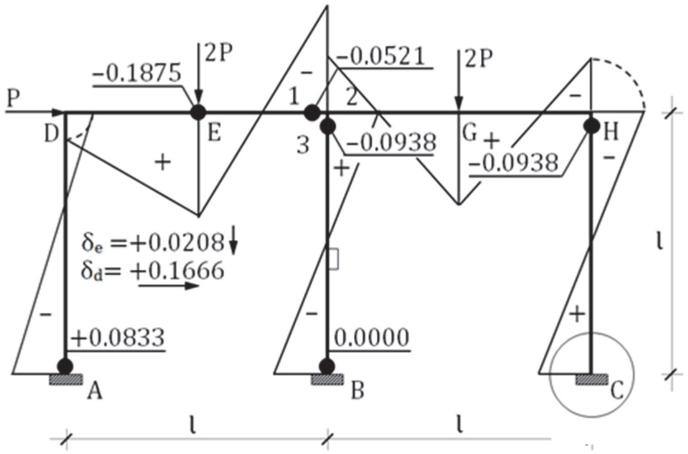

Figure 3: Hypothesis of plastic hinge at node C.

The plastic moments of the elements are assumed equal to $2M_p$ for beams and M_p for pillars. The stiffness of the beams is fixed twice that of pillars. The stretched fibers of the elements are the internal ones except for the central pillar whose stretched fibers are indicated by a small rectangle. In figure the plastic hinges are localized together with the bending moment distribution at collapse. The multiplier at collapse is equal to $\lambda_p = 5.50M_p/Pl$. These results have been obtained making use of the fundamental theorems of the Limit analysis. It is worth to note that, being the structure six times hyper-static, at collapse the number of plastic hinges is seven.
Hypothesis is made that the last formed plastic hinge is that at node C (Fig. 3). In the same figure the rotations at plastic hinges are reported. These were calculated through the solution of the system of equations obtained by applying the displacement method and taking into account the convention on signs at Section 4.
The horizontal displacement at node D (assumed positive if rightward) and the vertical displacement at section E (assumed positive if downward) are also reported, both clearly deducted by solution of the above-mentioned system. For rotations, the factor M_pl/EI is omitted, for displacements the factor M_pl/EI.
For a greater clarity, the diagram of moments has been superimposed. The analysis of these results shows that the choice of the last plastic hinge at node C is false.
It is in fact sufficient to observe that the rotation at node E is negative while the plastic moment at the same section has a positive sign.
On the basis of the criterion showed in Section 3, an articulated movement similar and concordant to the collapse motion has to be built. It is defined by a clockwise rotation α of the pillar AD (expressed for less than the factor M_pl/EI). This rotation, accordingly with the convention on signs in Section 4, gives to each plastic hinge a rotation well defined in value and sign, as indicated in Fig. 4. Under the hypothesis that the last plastic hinge finds at node C (Fig. 3), the rotations of plastic hinges are now summed to the rotations obtained through the impressed articulated motion (Fig. 4).

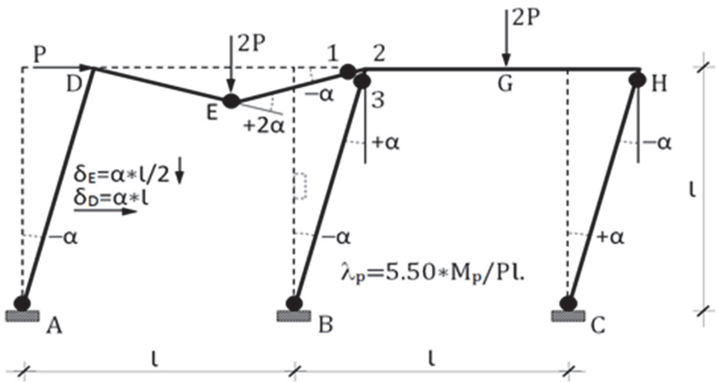

Figure 4: Scheme with an imprinted articulated motion.

In this way, the following sums are obtained:

node A $(+0.0833 \ -\alpha \ )$
node B $(\ 0.0000 \ -\alpha \ )$
node C $(\ 0.0000 \ +\alpha \ )$
node E $(-0.1875 \ +2\alpha)$
node 1 $(-0.0521 \ -\alpha \ )$
node 3 $(-0.0938 \ +\alpha \ )$
node H $(-0.0938 \ -\alpha \ )$

For the concordance on verses, each one of these sums must have a verse concordant to the corresponding plastic moment and this implies that, for each plastic hinge, their product must be always not negative. This condition implies that the following inequalities must be satisfied:

Node A	$(+0.0833 - \alpha)*(-M_p) \geq 0 \longrightarrow$	$\alpha \geq +0.0833*M_p$
Node B	$(+0.0000 - \alpha)*(-M_p) \geq 0 \longrightarrow$	$\alpha \geq 0.0000*M_p$
Node C	$(+0.0000 + \alpha)*(+M_p) \geq 0 \longrightarrow$	$\alpha \geq 0.0000*M_p$
Node E	$(-0.1875 + 2\alpha)*(+2M_p) \geq 0 \longrightarrow$	$\alpha \geq +0.0938*M_p$
Node 1	$(+0.0417 - \alpha)*(-2M_p) \geq 0 \longrightarrow$	$\alpha \geq +0.0417*M_p$
Node 3	$(-0.0938 + \alpha)*(+M_p) \geq 0 \longrightarrow$	$\alpha \geq +0.0938*M_p$
Node H	$(-0.0938 - \alpha)*(-M_p) \geq 0 \longrightarrow$	$\alpha \geq -0.0938*M_p$

plastic moments at collapse
articulated motion
plastic hinges rotation: last hinge in C

Each one of these conditions furnishes a lower limit for the parameter α. By selecting, among these, the greatest value, that is $\alpha=0.0938*M_pl/EI$, all the inequalities associated to the seven plastic hinges characterizing the collapse kinematics are simultaneously satisfied. Therefore, by substitution into the expressions of the above sums of rotations, the complete picture of all rotations is obtained. These are depicted in fig. 5 together with the diagram of bending moments at collapse. Remind is made that rotations are expressed less than the factor M_pl/EI.
The analysis of the result, as obvious, highlights everywhere the concordance in verses between plastic moments and rotations at correspondent hinges. It is also to be noted that, in the case under study, the last hinges are two, namely at the middle section E of the left beam and at the head section 3 of the central pillar. For a better understanding these have been individualized through a white circle. The procedure ends here, once the two plastic hinges have been found together with the rotations of other hinges and displacements, at an instant just before the collapse.
Regarding, it has been considered appropriate to perform some checks. In particular, the structure has been separately solved by applying the rotation method under three hypotheses: last hinge at section E, last hinge at the head section 3 of the central pillar; last hinges simultaneously at the two positions. In all the three cases the result coincided with that one above reported.

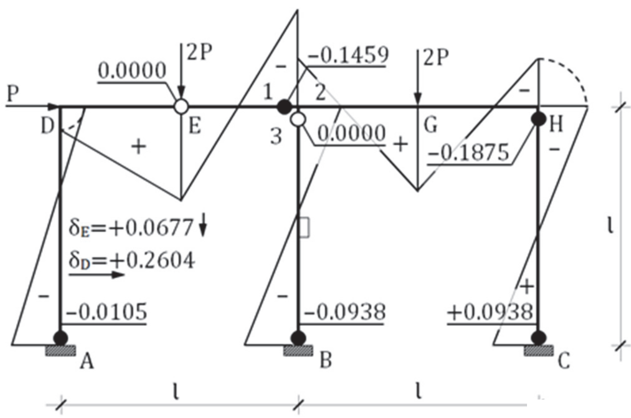

Figure 5: Collapse kinematic mechanism.

The frame has been further studied employing a step-by-step procedure: the same result was obtained together with the confirmation of the simultaneous appearance of the two plastic hinges in sections E and 3. Also, even the horizontal displacement at D node and vertical displacement at E node coincide with those found using this last procedure.

FINAL REMARKS

Apart its different theoretical structure, the procedure above developed, theoretically first and by means of an application on a double portal then, shows how much advantageous this new structure is from an operative point of view. According to the criteria developed in the previous monograph, in the procedure to localize the last hinge the number of systems to build and solve may result high (one for each hypothesis on the plastic hinge chosen as the last). In this new procedure, instead, it is sufficient to formulate a single hypothesis and to build only one system of equations whose solution permits to arrive to the desired result through a direct calculus. It is important to note that the hypothesis on the hinge chosen as the last one is not just an attempt (and therefore to be eventually repeated), but it is the first step of an algorithm that leads to the direct solution of the problem.

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