



Determination of the critical plane orientation depending on the fatigue curves for bending and torsion

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ABSTRACT. This paper contains a proposition setting out a new way of determining an orientation angle of the critical plane. The applied method involves an analysis of fatigue life results' scatter of several selected materials on the basis of a new criterion of multiaxial fatigue. The expression for the equivalent stress was derived on the basis of pure bending and pure torsion. The paper also contains a verification of the effectiveness of the proposed model by analyzing scatter bands and the value of the ratio of normal to shear stresses.

KEYWORDS. Critical plane; Fatigue life; Equivalent stress; Scatter analysys.



Citation: Kurek, M., Łagoda, T., Determination of the critical plane orientation depending on the fatigue curves for bending and torsion *Frattura ed Integrità Strutturale*, 41 (2017) 24-30.

Received: 28.02.2017

Accepted: 15.04.2017

Published: 01.07.2017

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INTRODUCTION

The literature contains a number of propositions including models for fatigue life assessment [1-4]. The work aimed at finding an algorithm for predicting fatigue life accompanying random loads was undertaken by Macha in 1979 [5], who applied the notion of stress for this purpose. Over the successive years, the propositions were improved and modified, and each of them accounted for one or a few aspects of the fatigue life. With the time passing, the strain model was established [6], followed by the one based on energy [7], which applied the parameter of strain energy density parameter acting on the critical plane. Despite the large number of propositions, there is still a need to modify the existing models. The justification for the adopted foundations is based on the scatter of the gained calculation results of the fatigue [8]. They form an important factor used for verifying the effectiveness of the proposed way of assessing fatigue strength. The objective of this paper is to present a new model applicable for assessment of fatigue life on the basis of a concept of the critical plane accompanied by the analysis of the scatter of the results.



THE ALGORITHM OF FATIGUE LIFE EVALUATION

The proposed algorithm of predicting fatigue life was formulated on the basis of a concept by Carpintieri [9]. The principal way of modifying the concept by Carpintieri involves a new relation that is established for the angle defining the critical plane orientation, as it plays a critical role in the assessment of fatigue strength.

Calculations were done using multiaxial fatigue criterion, based on the critical plane concept. The coefficients B and K in formulas for equivalent stress are calculated according to typical fatigue limits for pure bending and pure torsion. The general form of the equivalent stress [10] according to the criteria on critical plane is expressed as:

$$\sigma_{eq}(t) = B\tau_{\eta_s}(t) + K\sigma_{\eta}(t) \quad (1)$$

Authors use criterion (1) where weighing factors B and K [11] using pure bending and pure torsion can be defined as:

$$B = \frac{B_2 - \frac{\sin(90^\circ + 2\beta)}{\cos^2 \beta}}{\frac{\sin 2\beta \sin(90^\circ + 2\beta)}{2 \cos^2 \beta} + \cos(90^\circ + 2\beta)} \quad (2)$$

$$K = \frac{2 + B \sin 2\beta}{2 \cos^2 \beta} = 2 - \frac{\sigma_{af}}{\tau_{af}} = 2 - B_2 \quad (3)$$

$$B_2 = \frac{\sigma_{af}}{\tau_{af}} \quad (4)$$

$\sigma_{\eta}(t)$ is the normal stress and $\tau_{\eta_s}(t)$ is the shear stress, both acting on the critical plane:

$$\sigma_{\eta}(t) = \sigma_{xx}(t) \cos^2 \alpha + \tau_{xy}(t) \sin 2\alpha \quad (5)$$

$$\tau_{\eta_s}(t) = -\frac{1}{2} \sigma_{xx}(t) \sin 2\alpha + \tau_{xy}(t) \cos 2\alpha \quad (6)$$

where

$$\alpha = \alpha_{\eta} + \beta \quad (7)$$

being α_{η} the angle defined by the direction of the maximum normal stress if the above damage accumulation method is applied and β is

$$\text{ctg}(4\beta) = 22.5 \left[\left(\frac{1 + \sqrt{3}}{2} \right) - \frac{\sigma_a(N_{fi})}{\tau_a(N_{fi})} \right] = 22.5 \left[\frac{1 + \sqrt{3}}{2} - B_2(N_{fi}) \right] \quad (8)$$

The proposition (8) was derived on the basis of the variability of calculated fatigue strength depending on angle β for several selected materials [11-13]. We can note here that formula (4) is based on the ratio of the fatigue life boundaries. In the proposition (8), this coefficient is relative to the number of cycles, and can be stated the following form

$$B_2(N_{fi}) = \frac{\sigma_a(N_{fi})}{\tau_a(N_{fi})} \quad (9)$$

A graphical interpretation of formula (8) is presented in Fig. 1, whereas the broken line marks the proposition developed by Carpintieri (10).

$$\beta = \frac{3}{2} \left[1 - \left(\frac{1}{B_2} \right)^2 \right] 45^\circ \rightarrow B(N_{fi}) = \frac{3}{2} \left[1 - \left(\frac{1}{B_2(N_{fi})} \right)^2 \right] 45^\circ \quad (10)$$

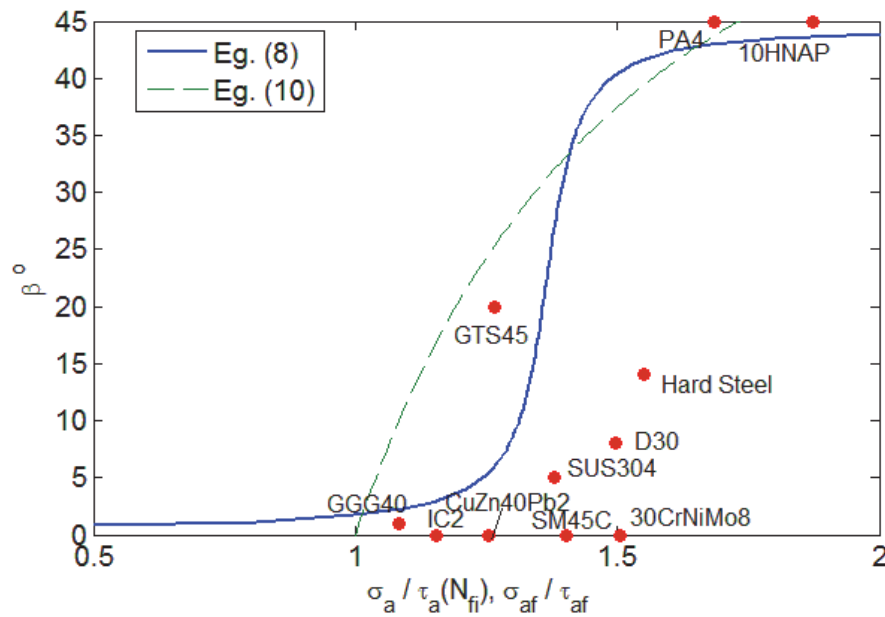


Figure 1: The relationship between the angle β and the parameter B_2 according to different models.

The final step is the calculation of the fatigue strength. For constant amplitude cyclic loading, the fatigue strength is evaluated by using Basquin's fatigue characteristics (A and m) in compliance with the relevant ASTM standard [14]. The formula for strength calculation under cyclic loading is expressed as follows:

$$N_{cal} = 10^{A - m \lg \sigma_{eq,a}} \quad (11)$$

where $\sigma_{eq,a}$ is the amplitude of the equivalent stress related to the critical plane (Eq.(1)), A, m - coefficients of regression equation for oscillatory bending.

FATIGUE STRENGTH SCATTER ANALYSIS

The proposition in Eq. (8) presented in Fig. 1 was developed on the basis of the results of a study involving several materials and requires verification. An analysis of the scatter of the results of fatigue life depending on the proposed angle and value of the ratio of the normal to shear stress was undertaken for this purpose. The analysis applied the values of the scatter bands for the particular materials just as it was calculated in [8] for angle β in the range $\langle 0^\circ, 45^\circ \rangle$. Fig. 2 presents the methodology followed during the analysis of the proposed expression (8) with regard to the relation of angle β and scatter T . The values of angles β were calculated so that the scatter T is in the range $\langle T_{min}, 1.1T_{min} \rangle$. On this basis, $\beta_{opt} \in \langle \beta_{min}, \beta_{max} \rangle$.

Fig. 3 illustrates relationship between scatter values T and the angle β for D30 [15].

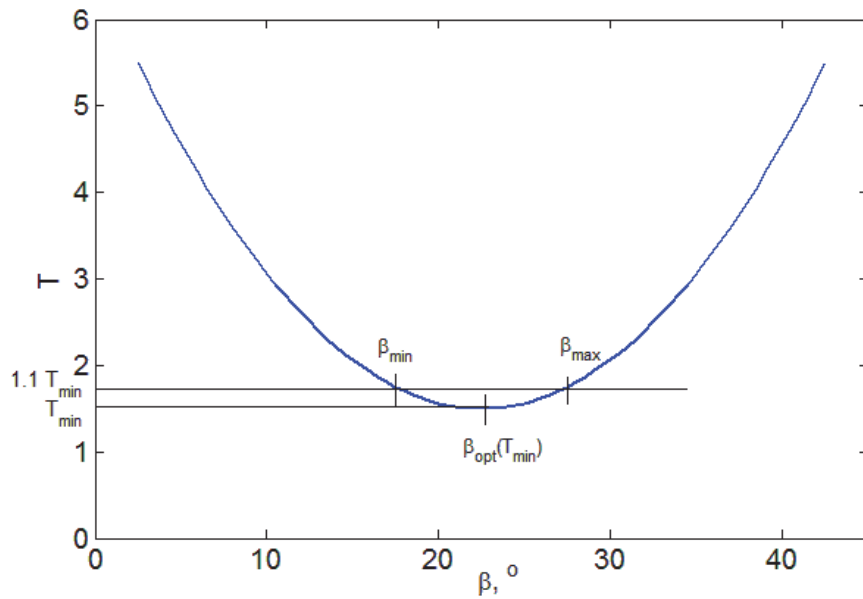


Figure 2: The methodology of presented calculations.

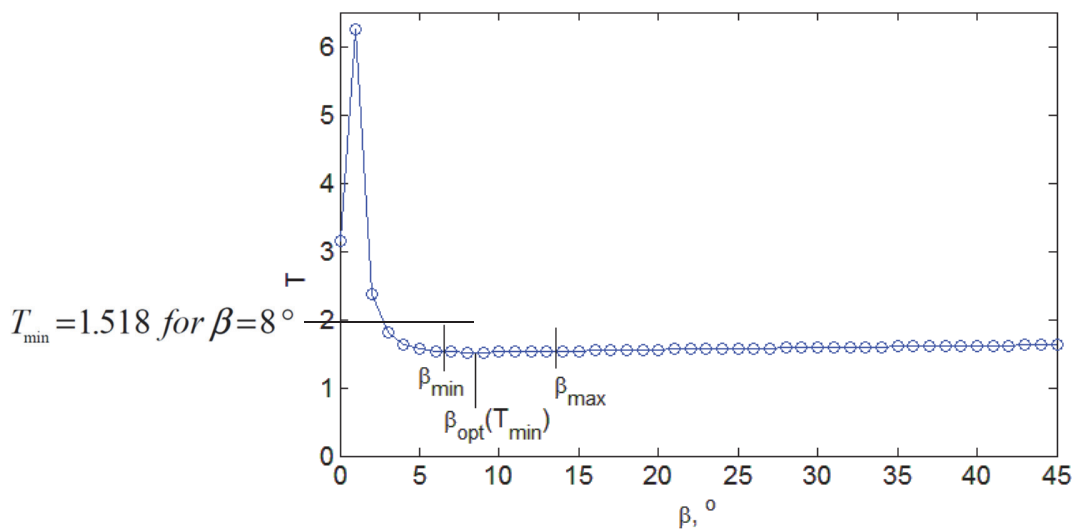


Figure 3: Relationship between scatter values T and the angle β for D30.

In addition, calculations were performed with regard to the relation of the ratio of normal stress to shear one depending on angle β . The study took into consideration the level of error of 10%. Accordingly, it was adopted that the ratio is established with the probability level of $\pm 5\%$

$$0.95 \frac{\sigma_a}{\tau_a} < \frac{\sigma_a}{\tau_a} < 1.05 \frac{\sigma_a}{\tau_a} .$$

One of the characteristics of fatigue tests is that the results demonstrate a considerable degree of scatter. This paper is based on an assumption that the error is at a level of 10%, which is sufficient during calculations of fatigue life with regard to loads. The results that were gained, were subsequently plotted onto the diagram representing the relation of angle β depending on the ratio of normal to shear stresses (Fig. 4).

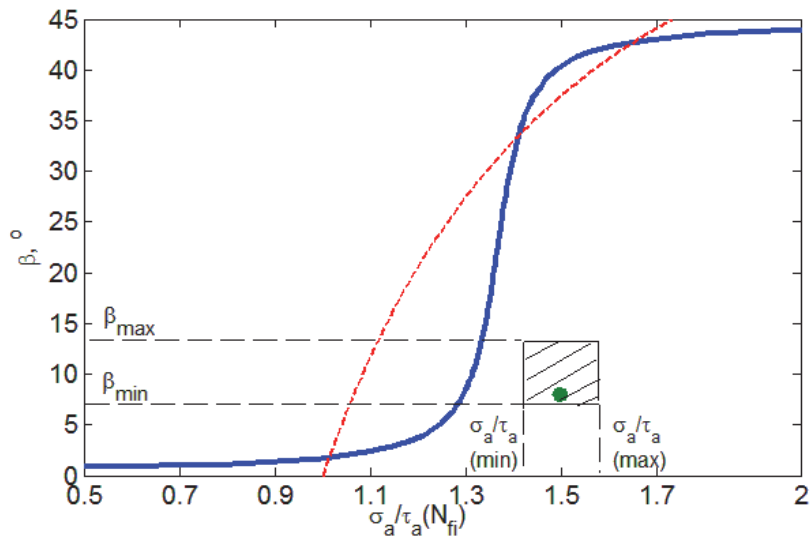


Figure 4: Exemplary analysis of material D30.

The hatched field is the area for which the results of the fatigue life are the same or differ insignificantly. Such analysis was performed for all test materials. Fig. 5 shows the distribution of the scatter results for GTS45 material [16].

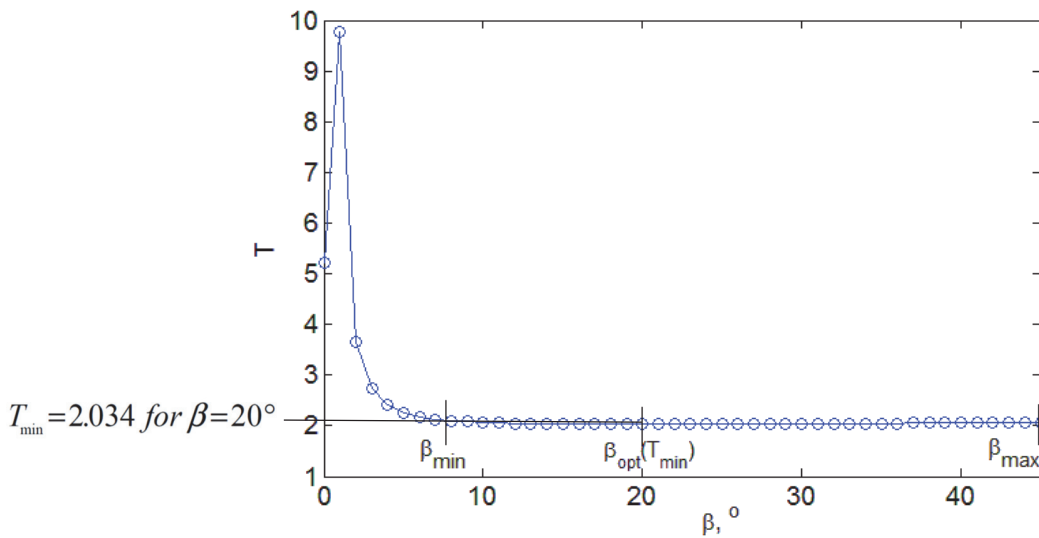


Figure 5: Relationship between scatter values T and the angle β for GTS45.

The subsequent analysis for exemplary materials is presented in Figs. 6-7.

From the analysis of Fig. 7, we can note down that the model also applies to materials, which are not covered by the proposed function (e.g. D30). In the future, it would be necessary to perform a more comprehensive static analysis involving the determination of fatigue characteristics with regard to pure bending and pure torsion for the respective materials.

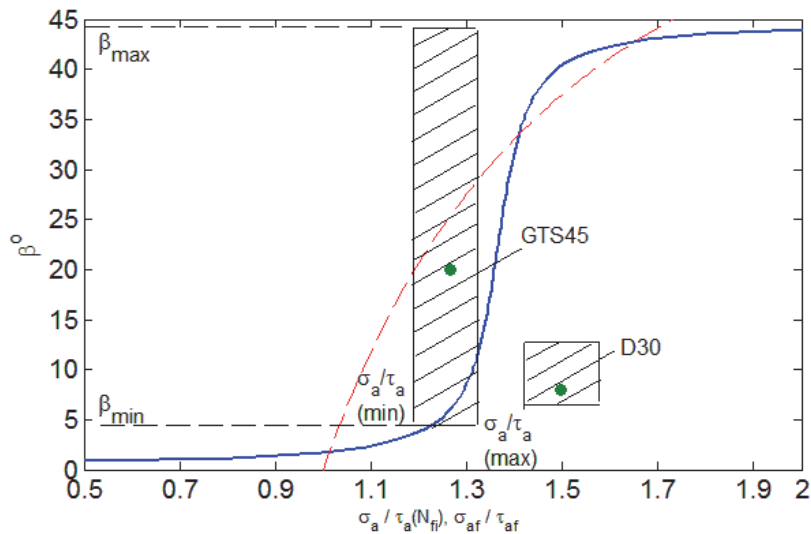


Figure 6: Exemplary analysis of selected materials.

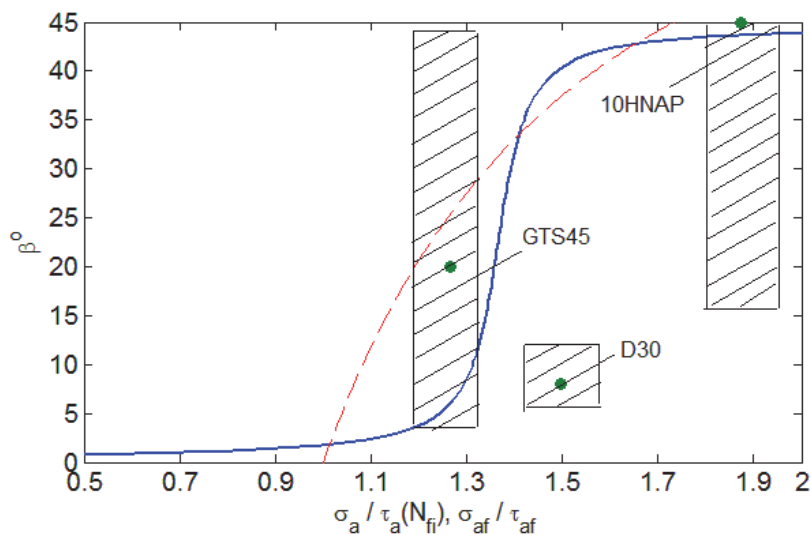


Figure 7: Exemplary analysis of selected materials.

CONCLUSIONS

On the basis of the analysis of the experiments as well as calculations and simulations, we can draw the following conclusions:

1. The proposed model has been developed on the basis of relations for materials in elastic-brittle state (Galileo's hypothesis) and in elastic-plastic state (HMH hypothesis). These states are nevertheless not limiting like in case of the Carpinteri's model.
2. The proposed model accounts for a wider range of the ratio of normal to shear stresses than the Carpinteri model.
3. The presented analysis is indispensable for the verification of the foundations and effectiveness of the new model.
4. Taking into consideration the scatter of the determined β angle and also the ratio of the fatigue limits for bending and torsion in may be noticed, that most of the materials coincide with the proposed model.
5. In the future more materials should be analyzed in order to continue the verification of the proposed model or to look for an even better suiting model for experimental tests.



ACKNOWLEDGMENTS

Project financed by the National Science Centre. Decision number: 2016/21/D/ST8/02007

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