On the applicability of Miner's rule for multiaxial fatigue life calculations under non-proportional load histories

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ABSTRACT. Fatigue design routines and computer codes must use some damage accumulation rule to deal with variable amplitude loadings (VAL), usually Palmgren-Miner (or Miner's) linear rule for lack of a clearly better option. Nevertheless, fatigue lives are intrinsically sensitive to the order of VAL events, which may e.g. induce residual stresses and thus change the critical point stress state, much affecting its subsequent residual life. On the other hand, in general, non-linear damage accumulation rules are not robust, resulting in better predictions than Miner's rule only for some specific load orders, requiring a case-by-case analysis. Therefore, Miner's linear damage rule still is the usual choice in practical fatigue calculations and assessments, giving reasonable predictions at least when properly combined with approaches that sequentially consider plasticity-induced effects, following the critical point stress/strain history in a cycle-by-cycle basis. In this work, Miner's rule is evaluated for non-proportional tension-torsion loadings on annealed tubular 316L stainless steel specimens. Normal-shear strain histories following either cross, diamond, circular or square paths are applied, and their fatigue lives are measured. Then, more complex VAL paths consisting of combinations of these individual path shapes are applied in other specimens, whose associated fatigue lives are predicted based on Miner's rule.

KEYWORDS. Miner's rule; Multiaxial fatigue; Non-proportional loading; Variable amplitude loading.

INTRODUCTION

A lthough practical fatigue analyses usually involve variable amplitude loads (VAL), material data (if available) normally is measured in constant amplitude load (CAL) tests. So, fatigue design routines must use some damage accumulation rule to deal with VAL. The classic linear damage accumulation rule, known as the Palmgren-Miner rule (or as Miner's rule) [1-2], predicts fatigue failures when the sum of the damage induced by each load event $\Sigma(D_i)$ equals the critical damage D_C the piece can sustain. D_C is usually arbitrarily defined as $D_C = 1 \Rightarrow \Sigma(n_i/N_i) = 1$, where n_i is the number of cycles of the i-th load event, and N_i is the number of cycles the piece would last if only that event loaded it. This heuristic hypothesis implicitly assumes that the various load events are independent.

Partial loss of available life is a simple way to quantify fatigue damage, but it is not the only one. As terminal fatigue failures occur at a critical crack size a_C, accumulated damage can be defined e.g. by the ratio $D = a/a_C$ in crack growth problems, where a is the current crack size.

Too many other semi-empirical rules are available to model fatigue damage accumulation under VAL. Some try to consider sequence effects not predictable by Palmgren-Miner (if it is used non-sequentially like a statistics), while others try to use less heuristic damage concepts. Fatemi and Yang [3] list 56 such rules, subdividing them into 6 groups: (i) linear damage evolution rules; (ii) bilinear damage evolution rules; (iii) rules that use modified fatigue life curves (SN or εN) to consider some sequence effects; (iv) Fracture Mechanics-based rules; (v) Continuum Damage Mechanics-based rules; and (vi) rules based on energy methods. Although damage accumulation modeling still is a fascinating research subject, none of such other (too many) rules got consensus to forever retire the old Palmgren-Miner rule, which has been used by structural engineers since 1924. Indeed, despite countless criticisms, it keeps on being by far the most used damage accumulation tool for fatigue design applications, bravely resisting to its poor performance with ordered loads, not to mention its lack of academic sophistication.

Continuum Damage Mechanics, in particular, is a modern technique that has many interesting features, among them be based on much more sound mechanical fundamentals that most of the concurrent heuristic rules. It associates different damage mechanisms to the apparent Young modulus' variation they cause in standard specimens, or to other similarly distributed parameters [4-5]. Therefore, it is no surprise it has been more useful to describe damage accumulation when it is caused by distributed damage mechanisms. However, its localized damage models, needed to describe most fatigue failures, still are at best controversial.

The value $D_C = 1$ usually chosen to quantify the critical damage is certainly arbitrary, since it is based on heuristic arguments, not on suitable mechanical foundations. Hence, it is no surprise to find $\Sigma n_i/N_i \neq 1$ in practical applications. Miner himself quoted $0.7 < \Sigma n_i/N_i < 2.2$ as typical values obtained in fatigue tests performed under random loads or load blocks [2]. Juvinall [6] affirms that usually $\Sigma n_i/N_i >> 1$ in two step loads when all small load events are applied before the large ones, whereas $\Sigma n_i/N_i << 1$ when all large load events act before the small ones, and that such ordered load blocks may have a huge dispersion, namely $0.18 < \Sigma n_i/N_i < 23$. However, as such well-ordered loading is rare in practice, for engineering purposes the linear damage accumulation rule produces reasonable and frequently conservative predictions in many fatigue design problems under real service loads, the main cause for its perennial popularity.

However, such a popularity does not mean that SN models applied with Miner's rule can safely solve all practical fatigue crack initiation assessment problems, even if applied to a rainflow-counted *representative* sample of the load history. In fact, fatigue lives are intrinsically sensitive to the *order* of VAL events, which may e.g. induce residual stresses and change the critical point stress state, much affecting its subsequent residual life. All statistics ignore sequence effects, but the Palmgren-Miner rule does not need to be used as so. Indeed, fatigue damage can be *sequentially* accumulated considering at least residual stress variations associated with cyclic yielding at every load event [7], using linear or fancier damage accumulation models.

To do so, ordered damage calculations by εN techniques may recognize such plasticity-induced effects, following the critical point stress/strain history in a cycle-by-cycle basis, and they can even be included in simpler SN calculation routines to quantify the effect of residual stress changes caused by high-load events that induce local yielding. This strategy can be numerically efficient when such macroscopic plasticity events are rare.

However, the order of the individual load events is also important even in the absence of macroscopic plasticity, probably due to its effect on localized microplasticity around the crack initiation point. This is an important issue when measuring Gassner curves obtained by applying sequences of load blocks composed by a series of CAL steps containing many cycles each. Indeed, even under the high cycle fatigue regime, it cannot be expected that the damage caused by a single ordered load block, with all its steps applied in a low → high → low sequence, is identical to the damage caused by the same steps,

but applied in a high → low → high order. In fact, such ordered loadings tend to generate very different fatigue lives, see Fig. 1 [8]. This figure also shows that the highest scatter in Miner's rule predictions happens at high mean stress values, indicating that load order effects are more important under higher stress levels.
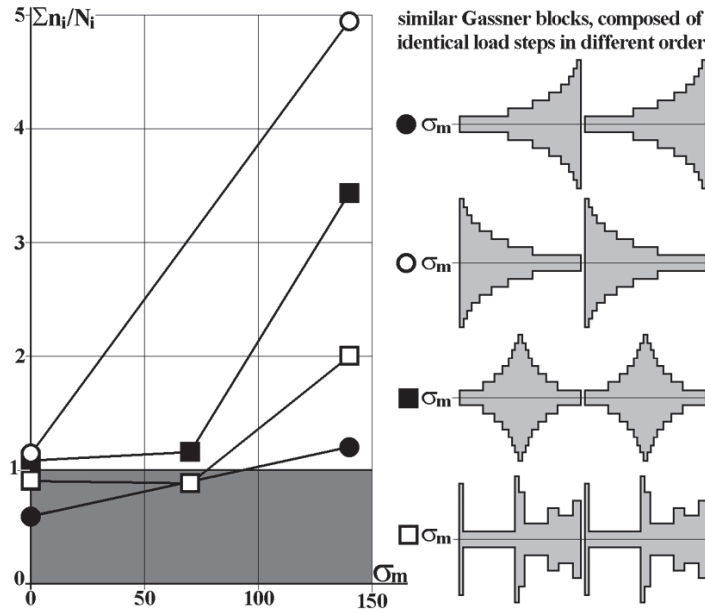

Figure 1: Step order and mean-stress effects on the lives of notched Al 7075-T6 test specimens with stress concentration factor 4, subjected to load blocks formed by the same load steps (these steps have the same mean stress σ_m, but practical spectra can produce steps with different σ_m) [8].

Therefore, it could be argued that one of the main reasons for the variability of the critical damage D_C is the load order effect associated with higher stress levels or overloads, which can induce residual stresses ahead of the initiating microcrack, affecting its subsequent fatigue life. As discussed before, if such load order effects are sequentially accounted for by εN techniques or even by da/dN short crack concepts, then Miner's rule should give predictions with improved reliability.

For instance, Elber's plasticity-induced crack closure idea [9-10] can be adapted to model the σ_m influence on uniaxial εN tests, assuming the fatigue damage process only continues after the microcrack is completely open. Indeed, Topper's group found they remain partially closed during part of their loading cycle. Hence, they assumed σ_m or σ_{max} effects on the so-called crack initiation phase would be *caused* by such crack opening loads, and proposed a new model to quantify these effects on εN curves [11-12]. In this way, fatigue damage would occur only on $\Delta\varepsilon_{eff}$, the effective portion of the hysteresis loop above the stress level σ_{op} that completely opens the microcrack at $\varepsilon_{min} + \Delta\varepsilon_{op}$, see Fig. 2.

$$\Delta\varepsilon_{eff} = \Delta\varepsilon - \Delta\varepsilon_{op} \qquad (1)$$

To precisely measure σ_{op} is no trivial task, but in many cases it can be assumed that $\Delta\varepsilon_{op}$ is approximately elastic, so

$$\Delta\varepsilon_{op} = \Delta\varepsilon - \Delta\varepsilon_{eff} \cong (\sigma_{op} - \sigma_{min})/E \qquad (2)$$

This effective strain range, which would be the cause for fatigue damage in the fatigue crack initiation stage, is illustrated in Fig. 2 as well. DuQuesnay et al. [12] proposed an empirical equation to estimate the opening stress σ_{op} in εN specimens, as a function of the applied maximum and minimum stresses, the cyclic yield strength S_{Yc}, and two material-dependent parameters α and β (which, for their 1045 steel, were $\alpha \cong 0.845$ and $\beta \cong 0.125$):

$$\sigma_{op} = \alpha \cdot \sigma_{max} \cdot \left[1 - \left(\sigma_{max}/S_{Yc}\right)^2\right] + \beta \cdot \sigma_{min} \qquad (3)$$
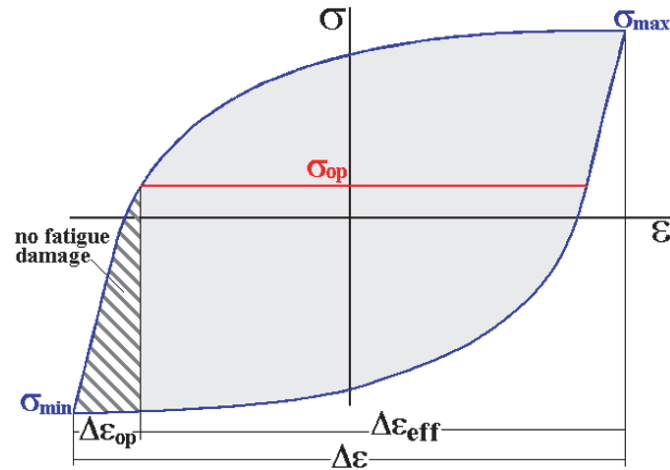
Figure 2: The effective strain range of the hysteresis loop, $\Delta\varepsilon_{eff} = \Delta\varepsilon - \Delta\varepsilon_{op}$, is the loop portion where the microcrack is completely open, thus can suffer fatigue damage.

The opening stress σ_{op}, which depends on σ_m and affects $\Delta\varepsilon_{eff}$, would be the physical cause for mean stress effects on fatigue crack initiation. Topper's tests showed that the ratio σ_{op}/σ_{max} can be negative, meaning microcracks can be entirely open even under a *compressive* load $\sigma_{op} < 0$, in particular under high stresses and negative stress ratios $R = \sigma_{min}/\sigma_{max}$. Therefore, it is interesting to rewrite Eq. (3) as a function of R as:

$$\sigma_{op}/\sigma_{max} = \alpha \cdot \left[1 - \left(\sigma_{max}/S_{Yc} \right)^2 \right] + \beta \cdot R \qquad (4)$$

If fatigue damage is caused by $\Delta\varepsilon_{eff} \cong \Delta\varepsilon - (\sigma_{op} - \sigma_{min})/E$, to calculate it using εN curves available in the literature, which associate $\Delta\varepsilon$ (not $\Delta\varepsilon_{eff}$) to the fatigue crack initiation life N, they should be properly adapted. To do so, Topper and co-workers proposed (and validated for some materials) an equation $\Delta\varepsilon_{eff} \times N$ that intrinsically includes the mean load effects:

$$\Delta\varepsilon_{eff}/2 = \varepsilon_L + \varepsilon_c'(2N)^{c'} \qquad (5)$$

This $\Delta\varepsilon_{eff} \times N$ curve is illustrated in Fig. 3, where ε_c' and c' are material properties, usually different from the equivalent Coffin-Manson's parameters, and ε_L is the strain fatigue limit, defined as the largest effective strain amplitude $\Delta\varepsilon_{eff}/2$ that does not cause fatigue damage in εN specimens.
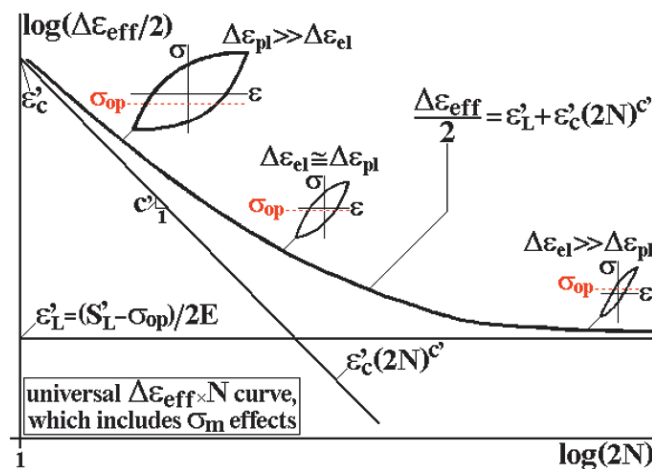

Figure 3: Universal $\Delta\varepsilon_{eff} \times N$ curve, which describes fatigue lives under any mean load.

The strain fatigue can be estimated from the traditional stress limit S_L under $R = -1$, assuming $\Delta\varepsilon_{eff}/2$ is purely elastic under infinite life and $S_L = \sigma_{max} << S_{Yc}$, hence:

$$\sigma_{op}/\sigma_{max} = \sigma_{op}/S_L \cong \alpha - \beta \Rightarrow \varepsilon_L \cong (S_L - \sigma_{op})/2E \cong (1 - \alpha + \beta)S_L/2E \tag{6}$$

Values of ε_c', c', and ε_L in Eq. (5) can be fitted from standard εN tests under $R = -1$, using Eq. (1)-(5) to calculate σ_{op} and $\Delta\varepsilon_{op}$ and convert $\Delta\varepsilon \times N$ into $\Delta\varepsilon_{eff} \times N$ data. Moreover, sparse periodic compressive underloads can be applied to conventional εN tests to guarantee that the initiated microcrack remains fully open, making $\Delta\varepsilon_{op} = 0$ and so $\Delta\varepsilon \equiv \Delta\varepsilon_{eff}$ [11]. After calibrated, these equations can be used to predict initiation lives under other stress ratios R, assuming mean load effects are caused solely by microcrack closure.

The expression for the microcrack opening load σ_{op} was obtained from fatigue tests under constant strain range $\Delta\varepsilon$. To use it for real service loads, DuQuesnay et al. assume the *smallest* σ_{op} value calculated during the entire variable amplitude load history remains invariable [12]. According to them, this hypothesis would be conservative, but it would produce fatigue damage predictions close to those measured experimentally. In this way, when $\sigma_{sup} \geq S_{Yc}$ is the largest and σ_{inf} is the smallest stress of the loading history, the smallest microcrack opening load would be given by σ_{op}^* where:

$$\sigma_{op}^* = \alpha \cdot \sigma_{sup} \cdot \left[1 - \left(\sigma_{sup}/S_{Yc}\right)^2\right] + \beta \cdot \sigma_{inf} \tag{7}$$

This idea can be used to evaluate the effects of tensile EP overloads on fatigue crack initiation, as well as the effects of compressive underloads that reduce σ_{op}^*, facilitating the opening of microcracks and increasing the effective strain range of subsequent load events, making them more harmful. Overloads and underloads are causes of very important load order effects on macrocrack fatigue propagation. According to this idea, the effective strain range $\Delta\varepsilon_{eff}$, calculated considering the smallest microcrack opening stress σ_{op}^* (or the value corresponding to e.g. the *0.5%* smallest) induced by the loading history, would be the parameter that quantifies mean load effects under real service loads. This method was qualified using standard εN specimens, but it might, at least in principle, be generalized for notched structural components [12].

DAMAGE ACCUMULATION IN MULTIAXIAL FATIGUE

There are several cycle-based models to quantify fatigue damage under multiaxial loading using some damage accumulation rule. Most of them can be separated into two classes, namely the invariant-based approach and the critical-plane approach, discussed as follows.

The *invariant-based* approach assumes fatigue damage is due to a Mises equivalent range, which mixes stress or strain components that act in all directions at the critical point. This approach assumes damage is due to a suitable combination of all stress components, so it is recommended for describing distributed-damage materials, or to model damage mechanisms such as multiple cracking in concrete, cavitation in ductile fracture, or fiber rupture in isotropic fiber-reinforced composites. In multiaxial fatigue applications based on this approach, the general Moment Of Inertia method [13] or some convex-enclosure method [14] must be applied after the use of a multiaxial rainflow count, to quantify load events and to obtain the associated stress or strain ranges, and then the corresponding damage using some invariant-based model such as Sines' or Crossland's. Then, the damage can be accumulated e.g. applying Miner's rule to combine the contributions of all multiaxial rainflow-counted load events.

The *critical-plane* approach, on the other hand, considers that fatigue damage is a truly local and directional problem, hence that only the most damaged plane of the critical point (or sometimes the plane experiencing maximum shear, depending on the fatigue damage model adopted) should be used to calculate fatigue lives. Therefore, it assumes that the critical plane of the critical point do not interact with the other planes, or else that there is no interaction among damage values eventually accumulated on planes that do not contain the fatigue crack. This approach should be preferred for describing fatigue damage in directional-damage materials, those which tend to fail by fatigue due to the formation and growth of a single dominant crack. This is the case of most metallic alloys, so the critical plane approach is very useful for practical applications. Moreover, this approach predicts both fatigue life and the orientation of the microcrack initiation plane.

Therefore, in the *critical-plane* approach, the stress or strain history must be projected onto several candidate planes at the critical point to identify the critical one. To predict the initiation of tensile microcracks along planes perpendicular to the free surface, a uniaxial rainflow count is applied to the projected normal stress or strain history to calculate accumulated Mode I damage e.g. using the multiaxial Smith-Watson-Topper (SWT) equation, or any other suitable model to describe tensile-sensitive materials [7]. Miner's rule could be then applied to accumulate the tensile damage of all events counted by the uniaxial rainflow.

For shear microcracks on planes perpendicular to the free surface, Miner's rule is applied instead to the accumulation of multiaxial fatigue damage using, e.g., Fatemi-Socie's model [7], a suitable model to describe shear-sensitive materials. It could be argued that both tensile and shear damage from a given plane should be added up using Miner's linear rule, however this would require e.g. that both SWT and Fatemi-Socie's models had been calibrated considering this combination of tensile and shear damage. Instead, in practice these models are calibrated only considering respectively the tensile or shear damage parameters, neglecting their interaction. Therefore, such an interaction should be disregarded in the subsequent predictions, to be coherent with the adopted model and its calibration routine. Miner's rule should thus be applied to either tensile or to shear damage on a given material plane, without adding them up.

To predict the initiation of shear microcracks along planes inclined 45° with respect to the free surface, the projected shear history must be rainflow-counted for each candidate plane. This counting must be done using a two-dimensional (2D) rainflow routine, e.g. a 2D version of the modified Wang-Brown rainflow [7] method, to combine non-proportional (NP) histories of in-plane and out-of-plane shear stresses τ_A and τ_B (or strains γ_A and γ_B). After this 2D rainflow, the 2D MOI or a convex enclosure method should be used in each rainflow-counted half-cycle to combine both shear stresses (or strains) into a path-equivalent shear range used in damage calculation with, e.g., Fatemi-Socie's shear-based damage model. Miner's rule can then used to obtain the accumulated damage on each candidate plane.

In summary, to properly describe fatigue damage under VAL conditions, all cycle-based multiaxial fatigue approaches require, in the end, some damage accumulation rule. Almost invariably, this is performed using Miner's linear rule because, in general, non-linear damage accumulation rules are not robust [3], resulting in better predictions than Miner's only for some load paths, but much worse for others.

However, most evaluations of Miner's rule are based either on uniaxial or on proportional multiaxial loading histories. Its applicability to non-proportional multiaxial load histories has not been much explored in the literature. However, NP out-of-phase histories have a very significant effect on fatigue damage when compared to proportional in-phase load histories. Under *strain control*, NP load histories are in general more damaging than proportional history paths with *same longest chord* L for two reasons: (i) the path-equivalent stress or strain range of the NP history is always larger than the proportional range L, as verified from the MOI [13] and convex enclosure methods [14], since longer load paths tend to induce more damage; and (ii) NP hardening, if present in the considered material (as in all austenitic stainless steels), tends to increase peak tensile normal stresses perpendicular to the critical plane.

On the other hand, under *stress control*, NP hardening could actually *decrease* the resulting strains, decreasing fatigue damage and thus competing with the path-equivalent effect. Hence, the fatigue analyst should be careful not to forget to consider in the calculations such different behaviors under strain or stress control, to avoid blaming Miner's rule for the scatter in the multiaxial damage predictions. As shown following, Miner's rule can be a very reasonable tool if the physics of the problem is properly understood and, in particular, if load-order plasticity effects are properly accounted for, either directly or indirectly.

EXPERIMENTAL VERIFICATION OF MINER'S RULE UNDER NP MULTIAXIAL LOADINGS

In this section, Miner's rule is evaluated for selected NP tension-torsion load histories with similar amplitudes. As discussed before, overloads or large differences in subsequent load amplitudes can induce significant load order effects, even in fatigue crack initiation problems. However, since the following analysis does not consider such effects, loading histories with similar amplitudes have been selected. The main objective of the following experiments is not to evaluate overload-induced effects, but to verify whether Miner's rule can give reasonable fatigue damage predictions not only for uniaxial and proportional, but also for NP multiaxial loadings.

The experimental evaluation uses complex 2D tension-torsion stress histories, applied on annealed tubular 316L stainless steel specimens in a tension-torsion servo-hydraulic testing machine, see Fig. 4. The experiments consist of strain-controlled tension-torsion cycles applied to six tubular specimens, each one of them following one of the six periodic $\varepsilon_x \times \gamma_{xy}/\sqrt{3}$ histories from Fig. 5. All tubular specimens had 30 mm outside diameter and 2mm cylindrical wall, to avoid significant strain gradients. The strains have been measured by a commercial tension-torsion clip-gage.

The individual amplitudes of each ε_x and $\gamma_{xy}/\sqrt{3}$ strain component are 0.6% for all six experiments. The first four experiments involve basic load path shapes: cross, diamond, circle and square, see Fig. 5. The last two involve combinations of the previous four: the square/cross path load blocks consisting of one square cycle followed by one cross cycle, while each load block of the square/circle/diamond path involved one square, one circle and one diamond cycle, in that order.

All the tests were carried out until a small crack was detected on the surface by visual inspection. In all specimens, the initiated crack was later confirmed to have surface widths between 1 and 2 mm. This variability contributes to the uncertainties in the experimental data, even though it can be inferred that the number of growth cycles between 1 and 2mm should be relatively small, since the visual inspection was carried out on a frequent basis. Tab. 1 shows the observed fatigue lives in number of blocks, where each block consists of a full load period.
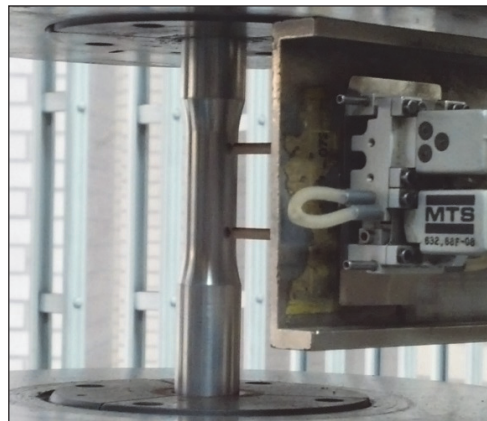

Figure 4: Tension-torsion testing machine and extensometer mounted on a tubular specimen.
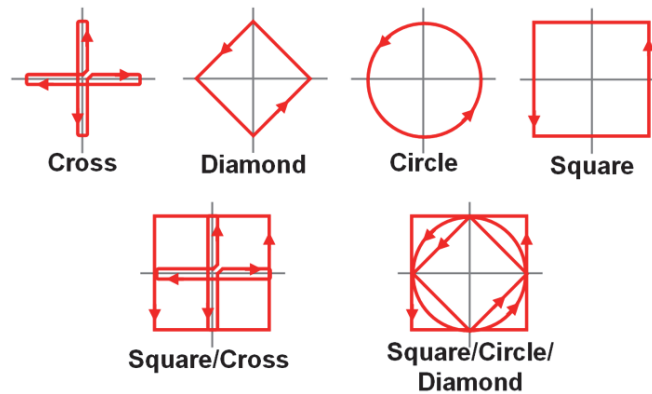

Figure 5: Applied periodic $\varepsilon_x \times \gamma_{xy}/\sqrt{3}$ strain paths on six tension-torsion tubular specimens, all of them with normal and effective shear amplitudes *0.6%*.

Tension-Torsion path	observed life (blocks)	Miner's prediction (blocks)
Cross	1535	-
Diamond	976	-
Circle	837	-
Square	772	-
Square/Cross	342	514
Square/Circle/Diamond	288	285

Table 1: Observed initiation lives, in number of blocks, for each applied tension-torsion path, and Miner's predictions.

Tab. 1 also shows Miner's rule *predictions* for the square/cross and square/circle/diamond paths, calculated from the observed lives of the four specimens subjected to the cross, diamond, circle and square paths. For the square/cross path, Miner's prediction for the number of blocks B is such that $1/B = 1/772 + 1/1535$, giving $B = 514$ blocks, an error of approximately 50%. Besides the usual scatter in fatigue (due to undetected microscopic defects) and of Miner's rule predictions, this error might also be attributed to the difficulty to detect a small crack on the surface, which was used as the initiation criterion to measure the number of blocks. Moreover, the critical plane of the square, cross and square/cross path specimens, where the microcrack initiates, could be significantly different, requiring the application of the critical plane approach to account for this and only then apply Miner's rule.

For the square/circle/diamond path, Miner's prediction is such that $1/B = 1/772 + 1/837 + 1/976$, giving $B = 285$ blocks, an unusually small prediction error of only 1%. This result is reassuring towards the continued use of Miner's rule, at least for such NP loading paths with similar stress levels and amplitudes. But since Miner's rule is not a physical law, it can still result in significant prediction errors for some particularly ordered histories, or in variable amplitude histories with large variations in stress or strain amplitude.

CONCLUSIONS

L oad order effects can be very important in crack initiation, and must be considered to properly account for residual stress effects and micro/macro plastic memory in general, especially under low-cycle conditions. Nevertheless, Miner's rule can be applied for both high and low-cycle fatigue, as long as any significant plasticity effect is considered, in the original order it was applied. It was found that Miner's rule provides reasonable predictions for selected non-proportional tension-torsion histories, at least when the variable amplitude loading cycles have equivalent amplitudes that do not differ too much from each other. Finally, be aware that, in general, non-linear damage accumulation rules are not robust; therefore, Miner's linear damage rule still is the best choice in multiaxial fatigue calculations, giving accurate predictions when combined e.g. with the critical-plane approach.

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