



Estimation of fatigue lifetime for selected metallic materials under multiaxial variable amplitude loading

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ABSTRACT. This paper initially investigates the accuracy of two methods, i.e., the Maximum Variance Method (MVM) and the Maximum Damage Method (MDM), in predicting the orientation of the crack initiation plane in three different metallic materials subjected to multiaxial variable amplitude loading. According to the validation exercise being performed, the use of both the MVM and the MDM resulted in a satisfactory level of accuracy for selected three metals. Subsequently, three procedures to estimate the fatigue lifetime of metals undergoing multiaxial variable amplitude loading were assessed quantitatively. Procedure A was based on the MDM applied along with Fatemi-Socie's (FS) criterion, Bannantine-Socie's (BS) cycle counting method and Miner's linear rule. Procedure B was based on the MVM, FS criterion, BS cycle counting method and Miner's linear rule. Procedure C involved the MVM, the Modified Manson Coffin Curve Method (MMCCM), the classical rainflow cycle counting method and Miner's linear rule. The results show that the usage of these three design procedures resulted in satisfactory predictions for the materials being considered, with estimates falling within an error band of three.

KEYWORDS. Multiaxial fatigue; Variable amplitude loading; Critical plane; Life prediction.

INTRODUCTION

Since the beginning of the last century, devising a sound method to estimate the fatigue lifetime of a component subjected to variable amplitude (VA) multiaxial loading has been the goal of numerous experimental/theoretical investigations. There are four aspects that need to be considered to estimate fatigue lifetime under multiaxial variable amplitude fatigue loading, i.e., the cyclic stress-strain model, the cycle counting method, the damage model and the damage accumulation model. [1] In addition, also the following aspects should be considered under multiaxial variable amplitude load histories: determining the orientation of the critical plane and calculating the amplitude and mean value of the stress/strain components relative to the critical plane. [2]



Fatigue criteria based on the concept of the critical plane are generally considered to be more accurate for multiaxial fatigue life estimation [1]. As far the low/medium cycle fatigue regime is concerned, the most successful criteria are seen to be those proposed by Smith, Watson and Topper [3], Brown & Miller [4, 5], Fatemi & Socie [6], and Susmel [7, 8]. As to the determination of the orientation of the critical plane, the MVM and the MDM are widely discussed in Refs [9-11]. The MVM assumes that the damage in any material plane can be related to the variance of the stress/strain signal in that plane. The plane on which the variance of the resolved shear stain/stress reaches its maximum value is defined as the critical plane. The MDM postulates that the critical plane is that material plane which experiences the maximum extent of fatigue damage.

The rainflow cycle counting method [12] has been most widely and successfully used under uniaxial loading. Among the methods dealing with VA multiaxial loading histories, Bannantine and Socie’s (BS) method [13] and Wang and Brown’s method [14, 15] deserve to be mentioned explicitly.

Formalising an appropriate damage accumulation model is another tricky problem to be addressed properly in order to estimate fatigue damage under VA multiaxial loading [16,17]. Miner’s linear damage rule [18] is still the most used rule.

In this paper, the accuracy of the MVM and the MDM in predicting the orientation of the critical plane is assessed. The accuracy of three procedures suitable for estimating multiaxial fatigue lifetime of metallic materials is checked against experimental data taken from the literature. The considered design procedures are as follows:

- (a) Procedure A: FS criterion applied with MDM, BS cycle counting method and Miner’s linear rule;
- (b) Procedure B: FS criterion applied with MVM, BS cycle counting method and Miner’s linear rule;
- (c) Procedure C: MMCCM applied with MVM, rainflow counting method and Miner’s linear rule.

FATIGUE CRITERIA

FS criterion

Fatemi and Socie [6] proposed a shear-strain based multiaxial fatigue criterion that can be expressed as follows:

$$\frac{\Delta\gamma}{2} \left(1 + k \frac{\sigma_{n,max}}{\sigma_y} \right) = \frac{\tau'_f}{G} (2N_f)^{b_0} + \gamma'_f (2N_f)^{c_0} \tag{1}$$

where $\Delta\gamma/2$ is the shear strain amplitude relative to the critical plane, $\sigma_{n,max}$ is the maximum normal stress, k is a material constant, and σ_y is the material yield strength.

MMCCM criterion

The MMCCM [7, 8] postulates that the degree of multiaxiality and non-proportionality of the stress state at the critical location can be quantified through the following stress ratio:

$$\rho = \frac{\sigma_{n,m} + \sigma_{n,a}}{\tau_a} = \frac{\sigma_{n,max}}{\tau_a} \tag{2}$$

where τ_a denotes the shear stress amplitude relative to the critical plane, $\sigma_{n,m}$ and $\sigma_{n,a}$ are the mean value and the amplitude of the stress normal to the critical plane, respectively, and $\sigma_{n,max}$ is the maximal normal stress relative the critical plane.

For a given value of ρ , the profile of the corresponding modified Manson–Coffin curve can be described by using the following general relationship:

$$\gamma_a = \frac{\tau'_f(\rho)}{G} (2N_f)^{b(\rho)} + \gamma'_f(\rho) \cdot (2N_f)^{c(\rho)} \tag{3}$$

where $\tau'_f(\rho)$, $b(\rho)$, $\gamma'_f(\rho)$, and $c(\rho)$ are fatigue constants that can be determined from the fully-reversed uniaxial and torsional fatigue curves [7, 8].



BS CYCLE COUNTING METHOD

Bannantine and Socie [13] have proposed a method based on the critical plane concept and the rainflow cycle counting method. This method makes use of a major channel and some auxiliary channels. For those materials whose fatigue breakage is shear governed, the major channel is the shear strain history. For those materials characterised by a Mode I dominated cracking behaviour, the major channel is the normal strain history. The rainflow cycle counting method is used to post-process the master channel. The normal stress signal is the auxiliary channel for FS criterion. The schematic of BS cycle counting method is shown in Fig.1.

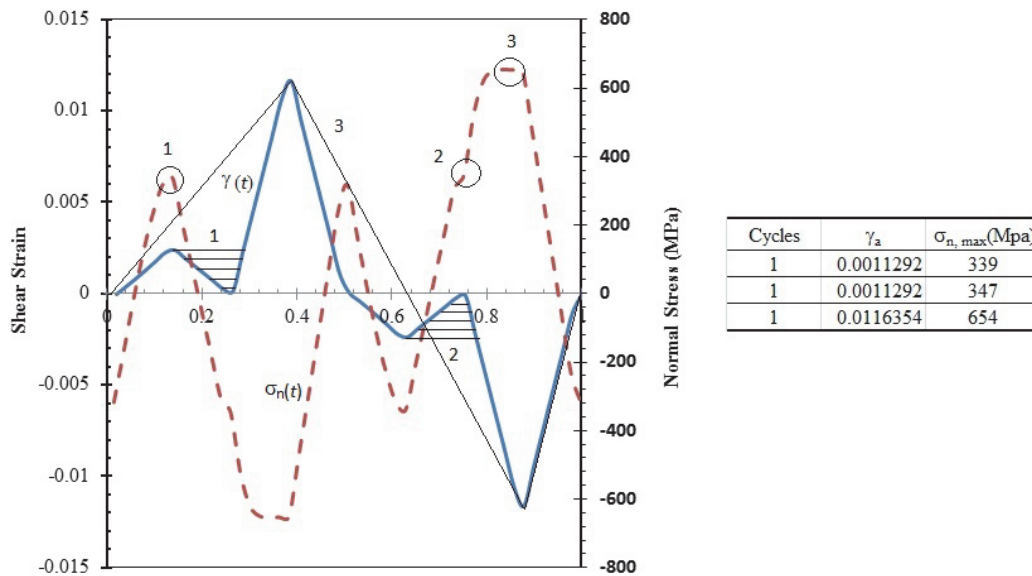


Figure 1: Schematic of BS cycle counting method.

EXPERIMENTAL EVALUATIONS

A number of experimental data were selected from the technical literature [19, 20] to check the accuracy of the considered procedures in estimating multiaxial fatigue lifetime. The summary of the static and fatigue properties of the investigated materials are reported in Tabs. 1 and 2. When the material constants listed in Tabs. 1 and 2 were not directly available in the original sources, they were estimated as follows [1]:

$$\tau'_f = \frac{\sigma'_f}{\sqrt{3}}; \gamma'_f = \sqrt{3}\epsilon'_f; b_0 = b; c_0 = c$$

The required stress component was calculated from the strain load histories being provided by using the model proposed by Jiang and Sehitoglu [21, 22]. The hardening effect under non-proportional loading was taken into account by making the following assumption [1]:

$$K'_{NP} = 1.25 \cdot K'; n'_{NP} = n'$$

Material	Ref.	E (GPa)	G (GPa)	σ_y (MPa)	k in FS
S45C	[19]	186	70.6	496	1
1050 QT steel	[20]	203	81	1009	0.6
304L stainless steel	[20]	195	77	208	0.15

Table 1: Static properties of the investigated materials

Material	Ref.	K' (MPa)	n'	ϵ'_f	σ'_f (MPa)	b	c	γ'_f	τ'_f (MPa)	b_0	c_0
S45C	[19]	1215	0.217	0.359	923	-0.099	-0.519	0.198	685	-0.12	-0.36
1050 QT steel	[20]	1558	0.123	2.01	1346	-0.062	-0.725	3.48	777	-0.062	-0.725
304L stainless steel	[20]	2841	0.371	0.122	1287	-0.145	-0.394	0.211	743	-0.145	-0.394

Table 2: Fatigue properties of the investigated materials

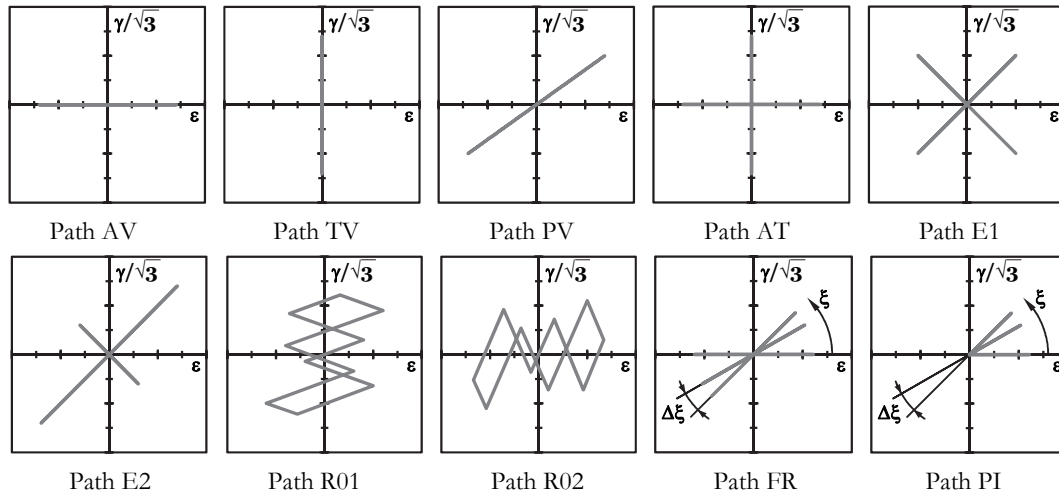


Figure 2: Investigated loading paths

The investigated loading paths are shown in Fig. 2. The stress and strain associated with any material plane can be obtained by coordinate transformation.

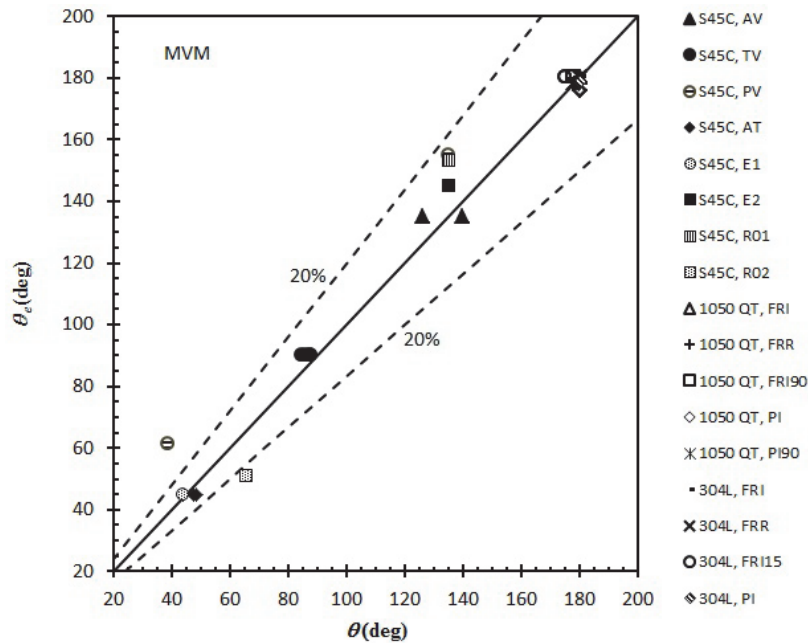


Figure 3: Comparison of observed and predicted orientation of critical plane by the MVM

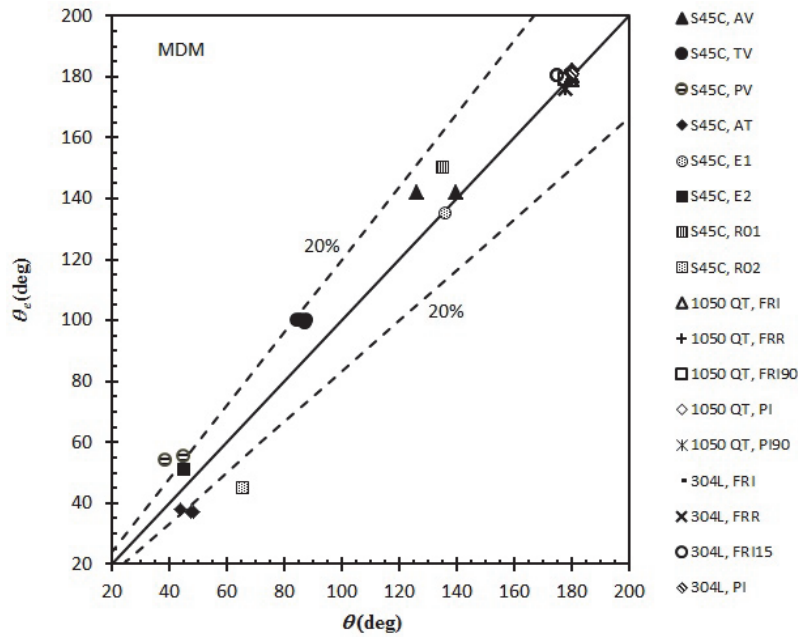


Figure 4: Comparison of observed and predicted orientation of critical plane by the MDM.

Critical plane orientation

The predicted orientation of the critical plane versus experimental orientation of Stage I crack plane for S45C steel, 1050 QT steel and 304L steel is reported in Figs 3 and 4. As it can be seen from these figures, the predictions made through the MVM and the MDM are characterised by the same level of accuracy, with 90% of the data falling within an error band of 20%.

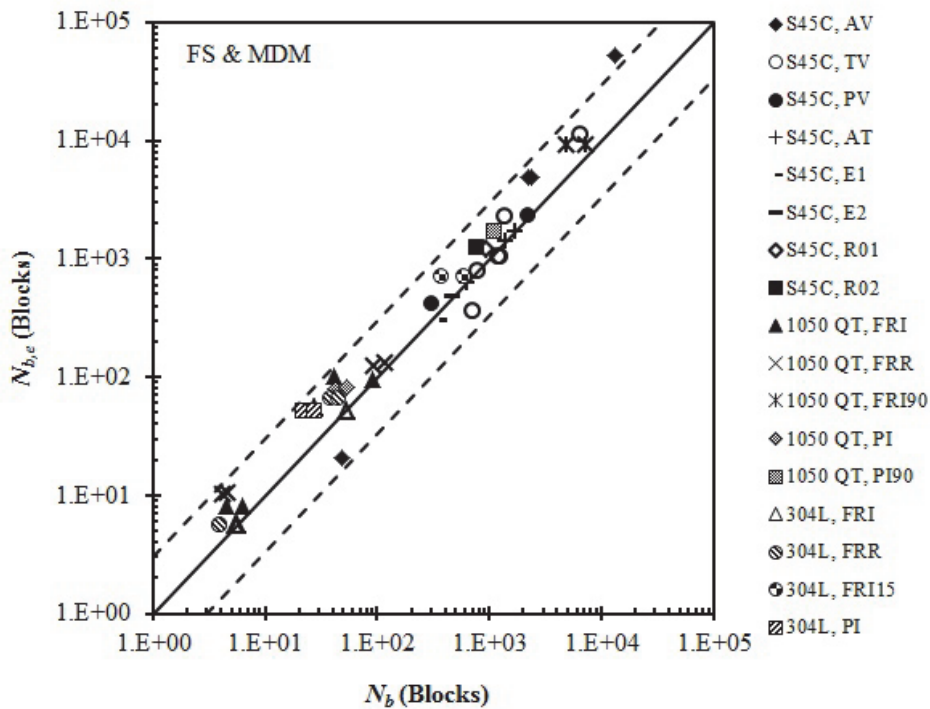


Figure 5: Comparison of observed and predicted fatigue lives by Procedure A.

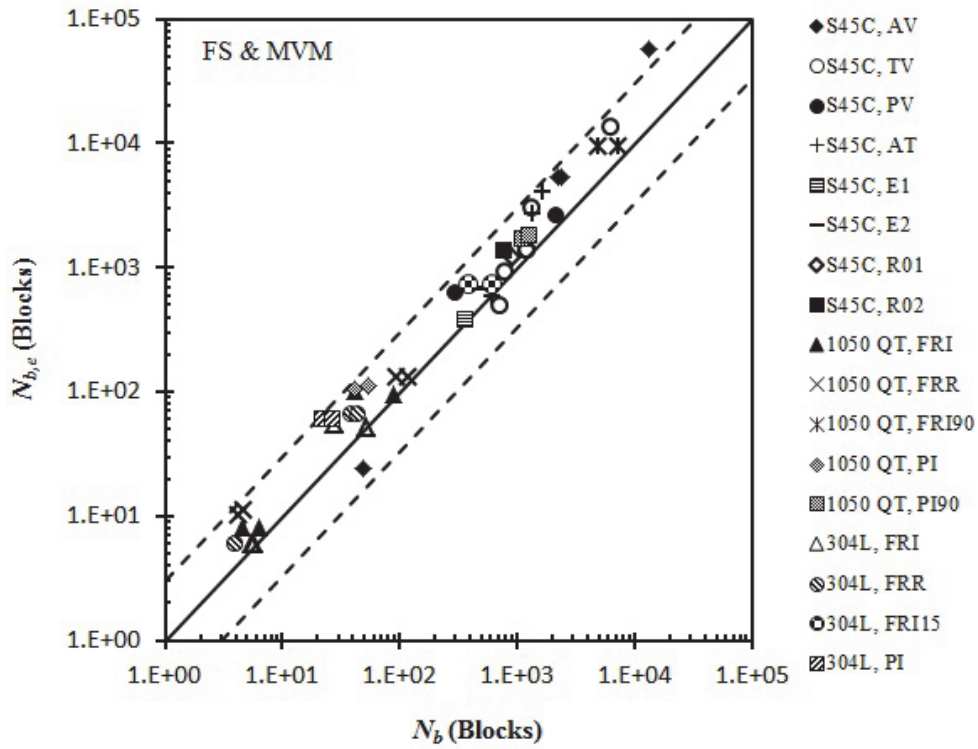


Figure 6: Comparison of observed and predicted fatigue lives by Procedure B

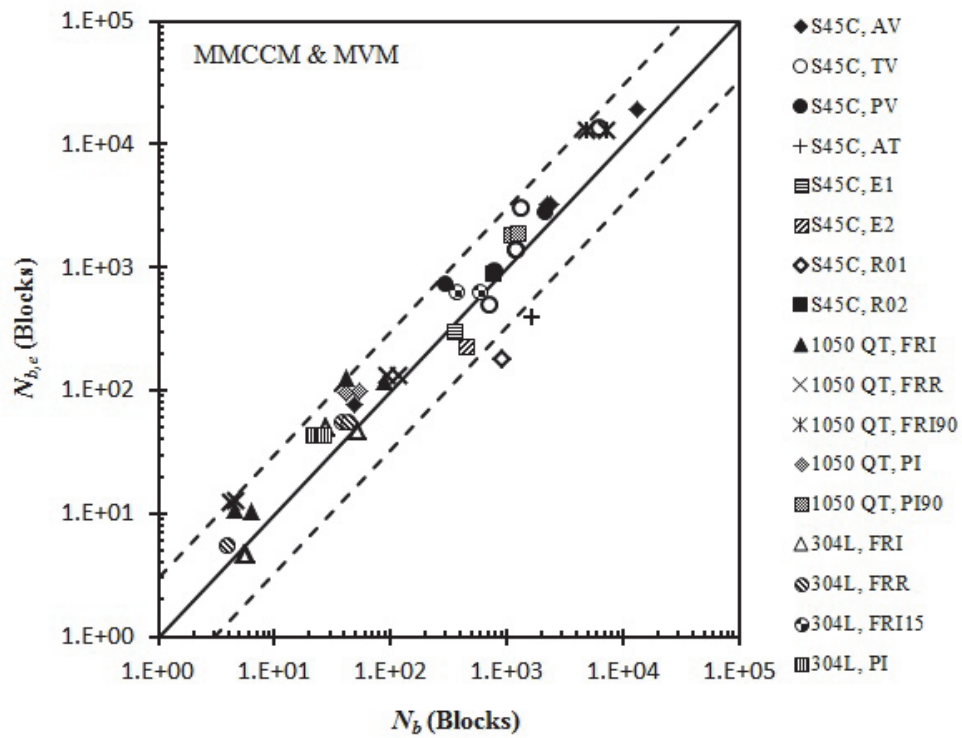


Figure 7: Comparison of observed and predicted fatigue lives by Procedure C



Fatigue lifetime prediction

The predicted versus experimental fatigue lifetime diagram determined via Procedure A is reported in Fig. 5. The predicted vs experimental fatigue lifetime diagram obtained through Procedure B is reported in Fig. 6. Finally, Fig. 7 shows the predicted vs experimental fatigue lifetime diagram determined using Procedure C. As it can be seen from Figs. 5, 6 and 7, all the data fall within an error scatter band of 3.

CONCLUSIONS

1. Both the MVM and the MDM can predict the orientation of the critical plane satisfactorily. The MVM is more efficient from a computation point of view.
2. Satisfactory fatigue lifetime predictions are obtained by using Procedure A, B and C.
3. The MVM can be applied with FS criterion successfully to predict fatigue lifetime for metallic materials undergoing VA multiaxial fatigue loading.

ACKNOWLEDGEMENTS

The Aviation Science Funds of China (No.: 2013ZA52008) and the National Natural Science Foundation of China (No.: 10702027) are acknowledged for supporting the present research work.

REFERENCES

- [1] Socie, D.F., Marquis, G.B., *Multiaxial Fatigue*, SAE, Warrendale, PA. (2000).
- [2] Wang, Y., Susmel, L., Critical plane approach to multiaxial variable amplitude fatigue loading, *Fracture and Structural Integrity*, 33(2015) 345-356.
- [3] Smith, K.N., Watson, P., Topper, T.H., A stress-strain function for the fatigue of metals, *J. Mater.*, 5(1970) 767-776.
- [4] [2] Brown M.W., Miller K.J., A theory for fatigue under multiaxial stress-strain conditions, In: *Proc institution of mechanical engineering*, 187 (1973)745-56.
- [5] Kandile, F.A., Brown, M.W., Miller, K.J., Biaxial low-cycle fatigue fracture of 316 stainless steel at elevated temperature, *Met. Soc. Lond.*, 280 (1982) 203-210.
- [6] Fatemi A., Socie D.F., A critical plane approach to multiaxial fatigue damage including out-of-phase loading, *Fatigue Fract Eng Mater Struct*, 11 (1988) 149-65.
- [7] Susmel L., Meneghetti G., Atzori B., A simple and efficient reformulation of the classical Manson-Coffin curve to predict lifetime under multiaxial fatigue loading-Part I: plain materials, *Journal of Engineering Materials and Technology*, 131 (2009) 021009-1-021009-9.
- [8] Wang, Y., Susmel, L., The modified Manson-Coffin Curve Method to estimate fatigue lifetime under complex constant and variable amplitude multiaxial fatigue loading, *Int. J. Fatigue*, 83(2016) 135-149.
- [9] Marciniak, Z., Rozumek, D., Macha, E., Verification of fatigue critical plane position according to variance and damage accumulation methods under multiaxial loading, *Int. J. Fatigue*, 58 (2014) 84-93.
- [10] Susmel L., A simple and efficient numerical algorithm to determine the orientation of the critical plane in multiaxial fatigue problems, *Int. J. Fatigue*, 32 (2010) 1875-1883.
- [11] Susmel, L., Tovo, R., Socie, D.F., Estimating the orientation of Stage I crack paths through the direction of maximum variance of the resolved shear stress, *Int. J. Fatigue*, 58 (2014) 94-101.
- [12] Matsuishi, M., and Endo, T., *Fatigue of metals subjected to varying stress*, Presented at Japan Society of Mechanical Engineers, Fukuoka, Japan, (1968).
- [13] Bannantine, J.A., Socie, D.F., A multiaxial fatigue life estimation technique, In: M.R. Mitchel, R.W. Landgraf (Eds.), *ASTM symposium on advances in fatigue lifetime predictive techniques*, ASTM STP 1122, American Society for Testing and Materials, Philadelphia, (1991) 249-75.
- [14] Wang C.H., Brown M.W., Life prediction techniques for variable amplitude multiaxial fatigue – part I: theories, *Trans. ASME, J. Eng. Mater. Technol.*, 118 (1996) 367–70.



- [15] Wang, C.H., Brown, M.W. Life prediction techniques for variable amplitude multiaxial fatigue – part II: comparison with experimental results, *Trans. ASME, J. Eng. Mater. Technol.*, 118 (1996) 371-374.
- [16] Fatemi, A., Yang, L., Cumulative fatigue damage and life prediction theories: a survey of the state of art for homogeneous materials, *Int. J Fatigue*, 20 (1998) 9–34.
- [17] Wang, Y., Zhang, D., Yao, W., Fatigue damage rule of LY12CZ aluminum alloy under sequential biaxial loading, *Sci. China – Phys., Mech. & Astro*, 57 (2014) 98-103.
- [18] Miner, M.A., Cumulative damage in fatigue, *J Appl Mech*, 67 (1945) AI59–64.
- [19] Kim, K.S., Park, J.C., Lee, J.W., Multiaxial fatigue under variable amplitude loads, *Trans ASME J. Eng. Mater. Technol.*, 121 (1999) 286-93.
- [20] Shamsaei, N., Fatemi, A., Socie, D.F., Multiaxial fatigue evaluation using discriminating strain paths. *Int. J. Fatigue*, 33 (2011) 597-609.
- [21] Jiang, Y., Sehitoglu, H., Modeling of cyclic ratcheting plasticity, Part I: development of constitutive equations, *Journal of Applied Mechanics*, 63 (1996) 720-725.
- [22] Jiang Y., Sehitoglu H., Modeling of cyclic ratcheting plasticity, Part II: comparison of model simulations with experiments, *Journal of Applied Mechanics*, 63 (1996) 726–733.