Two-parameter fracture model for cortical bone

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ABSTRACT. The analysis of the bone fracture behaviour is fundamental for prevention, diagnosis and treatment of traumas. In the present paper, an experimental analysis of the fracture behaviour of a bovine femoral cortical bone is carried out, where specimens are extracted from a diaphysis. Fracture toughness is computed by employing a two-parameter fracture model originally proposed for concrete. In order to take into account the possibility of crack deflection (kinked crack) due to osteons orientation, a modified version of such a model is here discussed. The fracture toughness results are then compared with those reported in the literature, related to a femur of an 18-month-old bovine.

KEYWORDS. Cortical Bone; Fracture Toughness; Quasi-brittle Material; Two-parameter Model.

INTRODUCTION

Bone is a specialised tissue which has both metabolic and mechanical functions [1-3]. The load-bearing capacity of bone is limited up to a certain extent, beyond which it fails [4]. The analysis of the fracture behaviour of bone is fundamental for prevention, diagnosis and treatment of traumas. Basic parameters which represent the structure and functions of bone have to be measured, such as its fracture toughness.

In the present paper, an experimental analysis of the fracture behaviour of a bovine femoral cortical bone is carried out, where specimens are extracted from a diaphysis. The experimental campaign is conducted to determine the fracture toughness, by employing a two-parameter fracture model originally proposed for concrete [5], that is, for a quasi-brittle material showing a nonlinear slow crack growth before the peak load is reached. In bones, such a behaviour is produced by the mechanism of extrinsic toughening categorised in four classes [6]: (i) constrained microcracking; (ii) crack deflection and twist; (iii) uncracked-ligament bridging; (iv) collagen-fibril bridging.

Such a two-parameter fracture method is based on experimental data obtained from three-point bending tests on single edge-notched specimens, and employs linear elastic fracture mechanics Mode I expressions. However, for the bone material, it cannot be applied in its original formulation since the crack starting from notch may deflect.

In order to understand the cause of such a deflection under Mode I loading (three-point bending), the bone microstructure level has to be briefly examined [7,8]. In cortical bone, it is represented by osteons (Fig. 1). The osteons

are oriented parallel to the bone axis, which consists of a vascular canal (or Haversian canal) surrounded by concentric lamellae. The interface between osteons and interstitial lamellae is called cement line (Fig. 1).
When the osteons alignment is perpendicular to the loading direction (transversal specimens), the stress state under three-point bending loading is biaxial due to normal stresses produced by bending and shear stresses at the cement line interface between osteons and interstitial lamellae [9]. In such a case, the crack starting from the notch is subjected to Mixed Mode loading (Mode I and Mode II). On the other hand, when the osteons alignment is parallel to the loading direction (longitudinal specimens), the stress state is uniaxial. In such a case, the crack starting from the notch is subjected to Mode I loading.
The Two-Parameter Model can be directly applied for the latter case, whereas a modified formulation is hereafter proposed for the former case in order to take into account that the crack is also subjected to Mode II loading. More precisely, the unloading compliance expression related to a single kinked crack is determined by employing the Castigliano theorem in the manner suggested by Paris [10], and the effective crack length is computed. Then, the critical stress-intensity factor at the crack tip is evaluated as a function of that related to a straight crack having length equal to the projected length of the effective kinked crack [11,12].

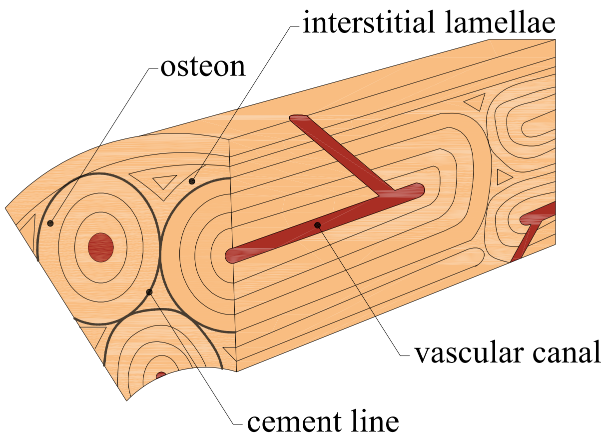


Figure 1: Cortical bone microstructure level.

Finally, the fracture toughness results here obtained are compared with those related to a femur of an 18-month-old bovine [13], experimentally determined according to the standard ASTM E399-1 [14].

TWO-PARAMETER MODEL

According to the Two-Parameter Model, the specimens present a notch in the lower part of the middle cross section (Fig. 2). The three-point bending tests are performed under crack mouth opening displacement control (average speed equal to 0.1 mm h^{-1}).

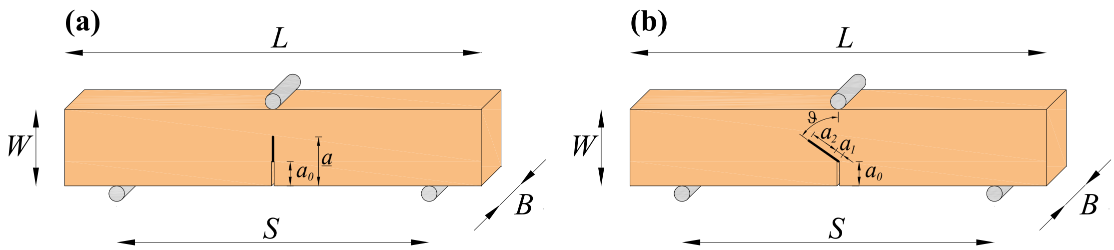


Figure 2: Crack propagates under: (a) pure Mode I; (b) Mixed Mode.

Each specimen is monotonically loaded: after the peak load is achieved, the post-peak stage follows and, when the force is equal to about 95% of the peak load, the specimen is fully unloaded. Then, the specimen is reloaded up to failure.

The initial compliance, C_i, is used to determine the elastic modulus, E [15]:

$$E = \frac{6S\,a_0 V(\alpha_0)}{C_i W^2 B} \tag{1}$$

where S, W and B are the loading span, depth and thickness of the specimen, respectively, a_0 is the notch length, and C_i is the linear elastic compliance. Further, the parameter V can be expressed as follows [15]:

$$V(\alpha_0) = 0.76 - 2.28\alpha_0 + 3.87\alpha_0^{\,2} - 2.04\alpha_0^{\,3} + \frac{0.66}{1-\alpha_0^{\,2}} \quad \text{with} \quad \alpha_0 = \frac{a_0}{W} \tag{2}$$

Therefore, if the crack propagates under pure Mode I, the effective critical crack length, $\underline{a}$, is determined from the following equation by employing an iterative procedure:

$$E = \frac{6S\,\underline{a}V(\alpha)}{C_u W^2 B} \tag{3}$$

where C_u is the unloading compliance, and $V(\alpha)$ is obtained from Eq. 2 by replacing a_0 with $\underline{a}$.
Finally, the Mode I critical stress-intensity factor, K_{IC}^S, is computed by employing the measured value of the peak load, P_{max} [15]:

$$K_{IC}^S = \frac{3P_{\max}S}{2W^2B}\sqrt{\pi\underline{a}}\, f(\alpha) \tag{4}$$

where:

$$f(\alpha) = \frac{1}{\sqrt{\pi}} \frac{1.99 - \alpha(1-\alpha)(2.15 - 3.93\alpha + 2.70\alpha^2)}{(1+2\alpha)(1-\alpha)^{3/2}} \quad \text{with} \quad \alpha = \frac{\underline{a}}{W} \tag{5}$$

MODIFIED TWO-PARAMETER MODEL

A modified procedure is hereafter proposed when crack propagates under Mixed Mode loading (Mode I and Mode II).
Specimens geometry and experimental test procedure are equal to those presented in the previous Section (see Fig. 2(b)). The elastic modulus is determined through Eq. 1.
Under Mixed Mode loading, the effective critical crack length, $\underline{a} = a_0 + a_1 + a_2$, is obtained from the following equation by employing an iterative procedure:

$$\begin{aligned} E = \frac{6S}{C_u W^2 B}\Big\{ & a_0 V(\alpha_0) + \\ & + \left[\cos^6\frac{\vartheta}{2} + \sin^2\frac{\vartheta}{2}\cos^4\frac{\vartheta}{2}\right]\left[(a_0 + a_1\cos\vartheta)V\Big(\frac{a_0 + a_1\cos\vartheta}{W}\Big) - a_0 V(\alpha_0)\right] + \\ & + \left[\cos^3\vartheta + \sin^2\vartheta\cos\vartheta\right]\Big[(a_0 + a_1\cos\vartheta + a_2\cos\vartheta)V\Big(\frac{a_0 + a_1\cos\vartheta + a_2\cos\vartheta}{W}\Big) + \\ & - (a_0 + a_1\cos\vartheta)V\Big(\frac{a_0 + a_1\cos\vartheta}{W}\Big)\Big]\Big\} \end{aligned} \tag{6}$$

Eq. (6) is deduced by employing the Castigliano theorem in the manner suggested by Paris [10], being ϑ the crack kinking angle (Fig. 2(b)) and $a_1 = 0.3a_0$. Note that, as is shown in Fig. 2(b), the kinked crack path consists of the two segment, named a_1 and a_2. If the value of a_2 obtained from Eq. 6 is negative, it means that the effective crack length is $\underline{a} = a_0 + a_1$ with $a_1 < 0.3a_0$. Such a length is obtained from the following equation by employing an iterative procedure:

$$E = \frac{6S}{C_u W^2 B}\left\{a_0 V(\alpha_0) + \left[\cos^6\frac{\vartheta}{2} + \sin^2\frac{\vartheta}{2}\cos^4\frac{\vartheta}{2}\right]\left[\left(a_0 + a_1\cos\vartheta\right)V\left(\frac{a_0 + a_1\cos\vartheta}{W}\right) - a_0 V(\alpha_0)\right]\right\} \tag{7}$$

Finally, the critical stress-intensity factor, K_{IC}^S, is computed through Eqs 4 and 5, by considering a straight crack having length equal to the projected length of the effective kinked crack:

$$\begin{aligned} &K_{IC}{}^S = \frac{3P_{\max}}{2W^2 B}\sqrt{\pi\left[a_0 + (a_1 + a_2)\cos\vartheta\right]}\; f(\alpha) \\ &\text{with } \alpha = \frac{a_0 + (a_1 + a_2)\cos\vartheta}{W} \quad \text{when } a_1 = 0.3a_0 \end{aligned} \tag{8a}$$

or

$$\begin{aligned} &K_{IC}{}^S = \frac{3P_{\max}}{2W^2 B}\sqrt{\pi\left[a_0 + a_1\cos\vartheta\right]}\; f(\alpha) \\ &\text{with } \alpha = \frac{a_0 + a_1\cos\vartheta}{W} \quad \text{when } a_1 < 0.3a_0 \end{aligned} \tag{8b}$$

RESULTS AND DISCUSSION

Values of fracture toughness, K_{IC}^S, of the analysed cadaveric femur diaphysis of a 24-month-old bovine are computed with respect to the osteons alignment: perpendicular or parallel to the loading direction. The former specimens are extracted from anterior (FA-1 and FA-2 in Tab. 1), posterior (FP in Tab. 1), medial (FM-1 and FM-2 in Tab. 1) and lateral (FL in Tab. 1) cortical bone, whereas the latter specimens from posterior cortical bone (FP-long1 and FP-long2 in Tab. 1). All specimens exhibit a non-linear slow crack growth before the peak load is reached.

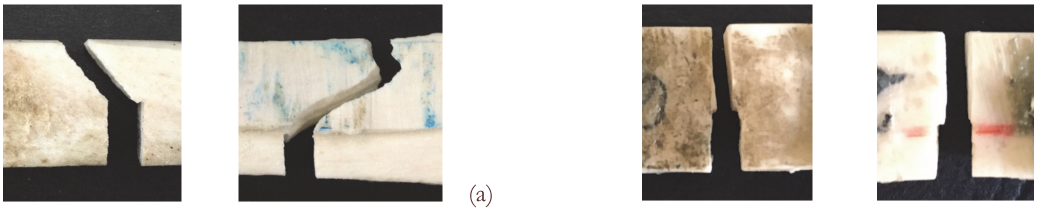
It can be observed that the fracture toughness values for specimens characterised by different osteons alignment with respect to the loading direction are significantly different. As a matter of fact, when the osteons alignment is perpendicular to the loading direction, crack grows under Mixed Mode (Fig. 3(a)), and a higher resistance to fracture is observed. When the osteons alignment is parallel to the loading direction, crack grows under Mode I (Fig. 3(b)), and a lower resistance to fracture is observed.

For transversal specimens, the average value of K_{IC}^S ($3.87 \pm 0.15 MPa\sqrt{m}$) is in the range of the cortical bone fracture values [16]. Such a value is then compared with those determined by Libonati et al. [13] according to ASTM standards [14], following the LEFM and considering cracks under pure Mode I loading. They found $K_{IC}^S = 5.6 \pm 0.1 MPa\sqrt{m}$ from SE(B) type and $K_{IC}^S = 5.8 \pm 0.6 MPa\sqrt{m}$ from C(T) type. The difference of such results with respect to those here obtained is due to the fact that the reduction of fracture toughness for a kinked crack as compared with the straight counterpart has not been taken into account by Libonati et al. in Ref. [13].

The present study highlights that the value of the near-tip stress-intensity factor of a kinked crack can be considerably lower than that for a straight crack of the same length, and that has to be taken into account for prevention, diagnosis and treatment of bone traumas.

Specimens No.	Elastic modulus E (MPa)	Fracture toughness K_{IC}^{S} (MPa m$^{1/2}$)
FA-1	13692.54	4.09
FA-2	14393.65	3.68
FP	14038.29	3.74
FM-1	14253.00	3.92
FM-2	13488.94	3.85
FL	14042.47	3.91
FP-long1	12371.99	2.14
FP-long2	12370.47	1.99

Table 1: Elastic modulus and fracture toughness.



(a) (b)

FA-1 FL FP-long1 FP-long2

Figure 3: Fracture: (a) Mixed Mode for transversal specimens FA-1 and FL; (b) pure Mode I for longitudinal specimens.

Conclusions

In the present paper, the behaviour of a compact bone in terms of fracture toughness has been analysed.
Fracture toughness has experimentally been evaluated through specimens obtained from the femur diaphysis of a bovine. The influence of local biaxial stress state on fracture behaviour has been analysed by employing two specimen types. As a matter of fact, when the osteons alignment is perpendicular to the loading direction (transversal specimens), the stress state is biaxial due to normal stresses produced by bending and shear stresses, at the cement line interface between osteons and interstitial lamellae. On the other hand, when the osteons alignment is parallel to the loading direction (longitudinal specimens), the stress state is uniaxial.
Then fracture toughness values have been computed by a modified version of the Two-Parameter Model originally formulated for crack propagating under Mode I. Such a modified version is here proposed for Mixed Mode (Mode I and Mode II). The theoretical results obtained are compared with some data available in the literature, by highlighting that the value of the near-tip stress-intensity factor of a kinked crack can be considerably lower than that for a straight crack of the same length.

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