Specimen thickness effect on elastic-plastic constraint parameter *A*

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ABSTRACT. Three-dimensional elastic-plastic problems for a power-law hardening material are solved using the finite element method. Distributions of the *J*-integral and constraint parameter *A* along the crack front for varying specimen thickness and crack depth are determined for edge cracked plate, center cracked plate, three point bend and compact tension specimens. The constraint parameter *A* is a measure of stress field deviation from the HRR field. Higher *A* values signify lower specimen constraint. Results of finite element analyses show that the constraint parameter *A* significantly decreases when specimen thickness changes from 0.1 to 0.5 of the specimen width. Then it has more or less stable value. Among four specimen the highest constraint is demonstrated by the compact tension specimen which has the constraint parameter *A* lower than its small scale yielding value.

KEYWORDS. Elastic-plastic crack tip field; Three-term elastic-plastic asymptotic expansion; Constraint parameter; Finite element method.

INTRODUCTION

The J-integral [1, 2] is the most used fracture mechanics parameter for structural integrity analysis of elastic-plastic cracked structures. However, fracture criterion based on the J-integral alone correctly works when the near crack tip stress fields are described by the one-term HRR asymptotic solution [3, 4]. In many cases (for example, short and inner cracks) a one-parameter approach is not suitable for fracture prediction. Finite element modeling shows that the one-term asymptotic expansion is unable to produce satisfactory description of near-tip stress fields in the microstructurally significant region. Even for the small scale yielding conditions the deviation of actual stress field from HRR-field is noticeable.

It is natural to assume that fracture in a structure occurs when a stress field in some region near the crack tip approaches the same value as in a test specimen under fracture load. Since the J-integral that controls the HRR-field cannot describe stresses in the crack-tip region under different load conditions it is necessary to utilize additional parameters and construct better equations for stress fields.

Betegon and Hancock [5] used elastic T-stress for studying effects on crack-tip triaxiality. While the T-stress can distinguish states with high and low constraint it cannot serve as a constraint parameter for elastic-plastic bodies because of its elastic nature.

O'Dowd and Shih [6] introduced a second fracture parameter in the form of a dimensionless stress Q which is defined as the difference between normal stresses in the near-tip region determined by a numerical analyses and the HRR stress field.

Main drawback of Q is its considerable dependence on radial and angular coordinates of a point selected for its determination.

A mathematical approach to the introduction of a second fracture parameter is based on higher order elastic-plastic asymptotic expansions of the stress field in the near crack tip region. Three-term asymptotic solutions have been reported by Yang, Chao and Sutton [7] and by Nikishkov [8, 9]. It was found that for Mode I plane strain crack the three terms of the asymptotic expansion are enough for representing the stresses in the crack tip region with sufficient accuracy. It appeared that the three-term expansion is controlled by just two parameters - the J-integral and an additional amplitude parameter A. Amplitude A can be used as a constraint parameter in elastic-plastic fracture. Two-parameter J - A fracture criterion has wider range of applicability than the criterion based on J-integral alone.

Here we present finite element three-dimensional elastic-plastic solutions for cracked specimens. Stress fields near the crack front are used for calculation of the constraint parameter A. Distributions of A along crack front are found for specimens of different thickness.

THREE-TERM ASYMPTOTIC EXPANSION

Suppose that the deformation behavior of an elastic-plastic material can be described with the Ramberg-Osgood uniaxial strain-stress curve:

$$\frac{\varepsilon}{\varepsilon_0} = \frac{\sigma}{\sigma_0} + \alpha\left(\frac{\sigma}{\sigma_0}\right)^n \tag{1}$$

where σ_0 is the yield stress, is the hardening coefficient, n is the hardening exponent ($n > 1$), $\varepsilon = \sigma_0 / E$, E is Young's modulus.

The three-term asymptotic expansion for the stress field near the tip of mode I crack in an elastic-plastic body can be presented in the following form [9]:

$$\frac{\sigma_{ij}}{\sigma_0} = A_0\rho^s\tilde{\sigma}_{ij}^{(0)}(\theta) - A\rho^t\tilde{\sigma}_{ij}^{(1)}(\theta) + \frac{A^2}{A_0}\rho^{2t-s}\tilde{\sigma}_{ij}^{(2)}(\theta) \tag{2}$$

Here σ_{ij} are stress components σ_r, σ_θ and $\sigma_{r\theta}$ in the polar coordinate system $r\theta$ with origin at the crack tip, $\tilde{\sigma}_{ij}^{(k)}$ are dimensionless angular stress functions obtained from the solution of asymptotic problems of order (0), (1) and (2). Angular stress functions $\tilde{\sigma}_{ij}^{(0)}$ and $\tilde{\sigma}_{ij}^{(1)}$ are scaled in such a way that maximal equivalent Mises stress is equal to unity. Power t is a numerically computed eigenvalue that depends on hardening exponent n. Power s is expressed as $s = -1/(n+1)$. Dimensionless radius ρ is defined by formula

$$\rho = \frac{r}{J/\sigma_0} \tag{3}$$

where J is the energy integral computed along small contour Γ_ε

$$J = \int_{\Gamma_\varepsilon}\left(Wn_1 - \sigma_{ij}\frac{\partial u_i}{\partial x_1}n_j\right)d\Gamma \tag{4}$$

Here W is the density of work done by stresses on mechanical strains, σ_{ij} are stresses, u_i are displacements, n_j and are components of external normal to the contour.

Coefficient A_0 is given by expression

$$A_0 = (\alpha\varepsilon_0 I_n)^s \tag{5}$$

where I_n is a scaling integral [3, 4].

While asymptotic series (2) contain three terms, the expression depends on two parameters - the J-integral which is hidden in radius ρ and the amplitude (or constraint) parameter A. The parameter A is a measure of stress field deviation from the HRR field.
Fracture criterion based on two parameters J and A compares J-integral values in a structure and in a test specimen that correspond to the same value of the constraint parameter A

$$J(P)|_A = J_C(A) \tag{6}$$

First, the J-integral value is computed for a structure subjected to load P. Then the constraint parameter A is estimated for the structure. Computed J-integral is compared to experimental fracture toughness corresponding to the same value of A.

CALCULATION OF *J*-INTEGRAL AND PARAMETER *A*

The equivalent domain integral method [10-12] is used for calculation of the J-integral values. This method replaces small contour integral by domain integral over large region.

$$J = -\frac{1}{f}\int_{V-V_\varepsilon}\left(W\frac{\partial q}{\partial x_1} - \sigma_{ij}\frac{\partial u_i}{\partial x_1}\frac{\partial q}{\partial x_j}\right)dV \tag{7}$$

Integration domain $V - V_\varepsilon$ around the crack segment is a difference between large cylinder V and small cylinder V_ε. Weight function q is selected in such a way that it is equal to zero on external and side surfaces of the integration domain. Area f under q-function on the inner surface of the integration domain is computed as

$$f = \int q\, dx_3 \tag{8}$$

An evident method for the determination of parameter A is solution of the quadratic Eq. (2) for any point near the crack front using finite element results for some stress component. Since finite element results are characterized by some scatter different points produce different A values.
Scattered data can be smoothed by the least squares fitting. The value of A for a set of points is determined by fitting of expression (2) to the finite element stress data at reduced integration points 2×2×2 in the near-crack front region which is significant for the local fracture process. Minimization of sum of squared differences between the finite element stresses and three-term asymptotic stresses leads to a cubic equation that can be solved by the direct method or by Newton's iteration.

NUMERICAL RESULTS

Elastic-plastic problems for a power-law hardening material are solved using the finite element method for the following specimens shown in Fig. 1: edge cracked plate (ECP), center cracked plate (CCP), three-point bend specimen (3PB) and compact tension specimen (CT). A typical finite element mesh for a quarter of an edge cracked plate $a/W = 0.3$ is shown in Fig. 2. It consists of 3008 quadratic 20-node elements and 13827 nodes. The crack front is surrounded by a polar mesh with 15 elements in angular direction. The radial size of the smallest element is $r_1 / W = 0.5 \cdot 10^{-4}$. Elements with smaller thickness are placed near the specimen surface.
Limit loads for rigid-plastic bodies are used to normalize the applied loads [13]:

ECP: $$\sigma_L = 1.455\sigma_0 \frac{a}{W}\left(-1+\sqrt{1+\left(\frac{W-a}{a}\right)^2}\right)$$

$$\text{CCP:}\quad \sigma_L = \frac{2}{\sqrt{3}}\sigma_0 \frac{W-a}{W}$$

$$\text{3PB:}\quad P_L = 1.455\sigma_0 \frac{(W-a)^2}{2H} \qquad (9)$$

$$\text{CT:}\quad P_L = 1.455\sigma_0 W\left(-1-\frac{a}{W}+\sqrt{2+2\left(\frac{a}{W}\right)^2}\right)$$

where σ_0 is yield stress, a is crack length, W and H are specimen width and semi-height. Limit loads for the edge cracked plate and center cracked plate σ_L are given if the form of remote stress. Limit load P_L for three point bend and compact specimens is a load per unit thickness.

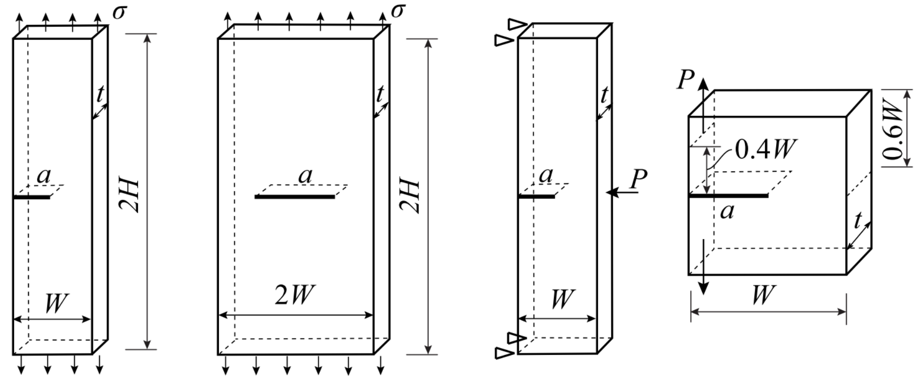

Figure 1: Specimens: edge cracked plate (ECP), center cracked plate (CCP), three-point bend specimen (3PB), compact tension specimen (CT).

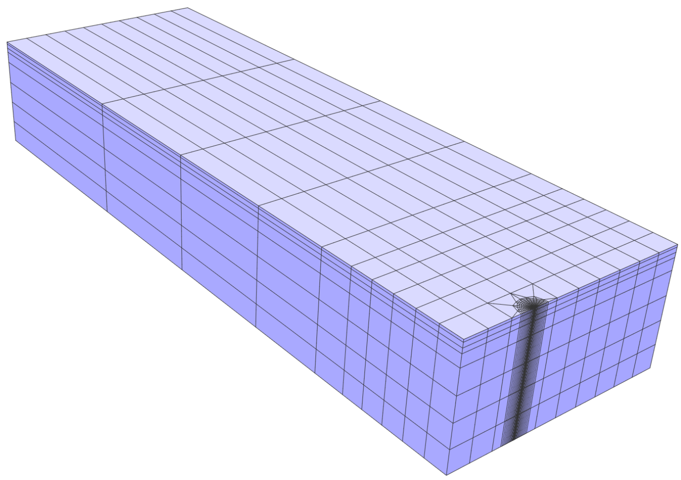

Figure 2: Finite element mesh for an edge cracked plate $a/W = 0.3$.

Values of the J-integral are calculated with the domain decomposition method for node locations at the crack front. Results for the J-integral are presented as normalized elastic-plastic stress intensity factors

$$\bar{K}_{ep} = \frac{1}{\sigma_0}\sqrt{\frac{JE}{\pi W(1-\nu^2)}} \qquad (10)$$

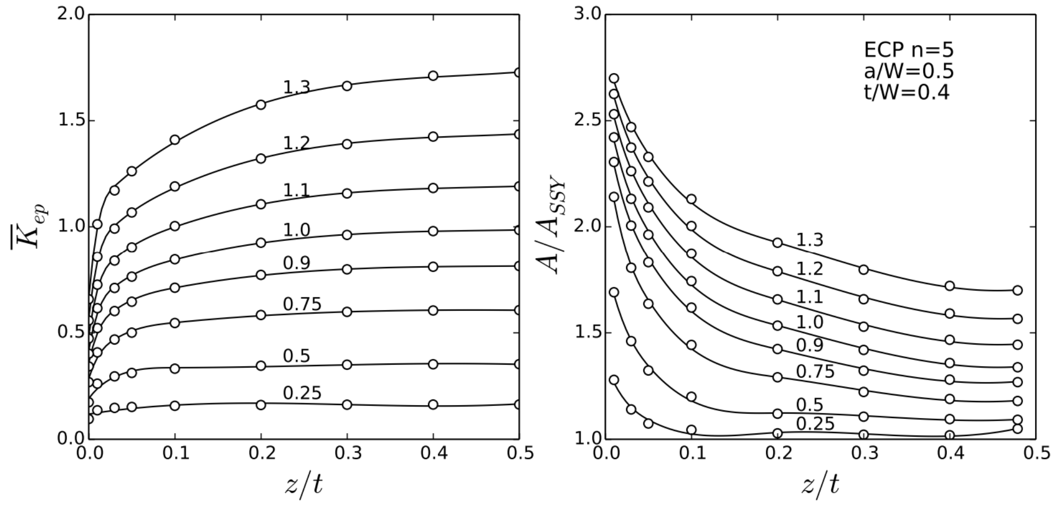

Figure 3: Distribution of elastic-plastic stress intensity factor K_{ep} and constraint parameter A along the crack front of the edge cracked plate ($n=5$, $a/W=0.5$, $t/W=0.4$)

Values of the amplitude A are determined by the least squares fitting procedure using circumferential stress σ_θ at integration points $2\times2\times2$ inside area $1<\rho<4$, $0<\theta<45^\circ$. According to the definition (2), the A values are dimensionless and normalized by a yield stress σ_0.

Normalization of the constraint parameter A is done with its small scale yielding value A_{SSY}. This value can be determined from solution of an elastic-plastic problem for any specimen under plane strain conditions loaded by infinitely small load. Another way (which is more efficient) is a solution of elastic-plastic plane strain crack problem with boundary conditions as stresses or displacements from elastic asymptotic distributions near the crack tip. For considered materials with hardening coefficient $\alpha=1$ and hardening power $n=5,10$ the small scale yielding values of the constraint parameter are $A_{SSY}(n=5)=0.380$ and $A_{SSY}(n=10)=0.184$.

A series of elastic-plastic finite element solutions has been performed with variation of the following parameters:

Specimens: ECP, CCP, 3PB, CT;
Hardening power: $n=5,10$;
Thickness: t/W from 0.1 to 1.0;
Crack depth: a/W from 0.1 to 0.7.

Typical results obtained after solution of an elastic-plastic problem are presented in Fig. 3 where elastic-plastic stress intensity factor K_{ep} and constraint parameter A along the crack front of the edge cracked plate for material with hardening power $n=5$, relative crack depth $a/W=0.5$ and relative thickness $t/W=0.4$ are given for load levels σ/σ_L from 0.25 to 1.3. Coordinate z is counted from a free specimen surface. While K_{ep} has its highest value at the specimen midplane the constraint parameter A considerably increases to the specimen surface. Higher values of A indicate lower constraint at the surface.

Dependencies of the constraint parameter A on specimen thickness t/W at the center of the crack front for all four specimens (ECP, CCP, 3PB, CT), hardening power $n=10$ and load level $P/P_L=1.0$ are shown in Fig. 4. General tendency is that the magnitude of the constraint parameter A decreases (higher constraint) with the increase of relative specimen thickness t/W. In most cases stabilization of the constraint parameter occurs for thickness $t/W>0.5$.

Change of the constraint parameter A with crack depth a/W at the center of the crack front for different specimens is presented in Fig. 5 for combination of parameters $n=10$, $t/W=0.5$, $P/P_L=0.75$. The center cracked plate shows low constraint for all crack depths. For the edge cracked plate constraint parameter A decreases with the increase of the crack depth and reaches its small scale yielding value for crack $a/W=0.7$. The three-point bend specimen has $A/A_{SSY}=1$ for crack depth around $a/W=0.5$ and less than unity for deeper cracks. Unique behavior is demonstrated by the compact tension specimen – the constraint parameter A is considerably lower A_{SSY} for all crack depths that are used for fracture toughness determination.

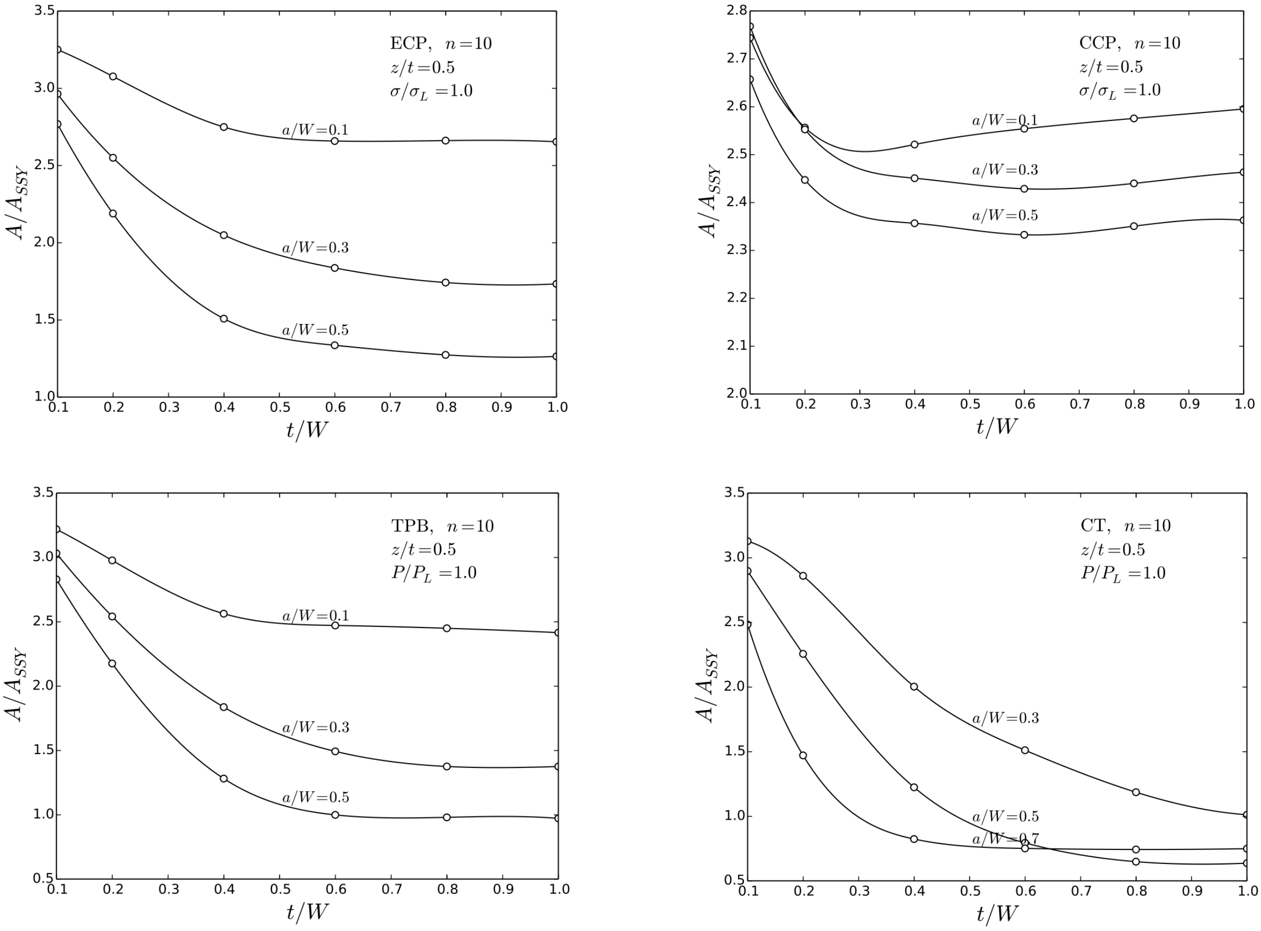

Figure 4: Dependence of constraint parameter A on specimen thickness t/W at the center of the crack front for ECP, CCP, TPB and CT specimens ($n=10$, $P/P_L=1.0$)

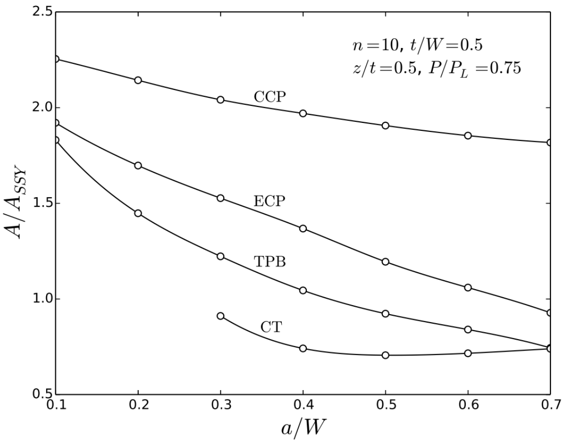

Figure 5: Dependence of constraint parameter A on crack depth a/W at the center of the crack front for different specimens ($n=10$, $t/W=0.5$, $P/P_L=0.75$).

Conclusions

Distributions of the constraint parameter A along the crack front were studied for specimens of different thickness. The constraint parameter A is amplitude for the second and third terms in the three-term elastic-plastic asymptotic expansion for the near-crack tip stress field. Three-dimensional elastic-plastic stress analyses of four specimens - edge cracked plate, center cracked plate, three point bend and compact tension specimens were performed using the finite element method with variation of specimen thickness and crack depth. Values of the constraint parameter A were determined by fitting stresses in the three-term asymptotic expansion to finite element results at integration points near the crack front. Higher values of the constraint parameter A show that the stress field is considerably deviates from the small scale yielding stress field that is usually called low constraint.

Typical distribution of the constraint parameter A is characterized by two features: minimal A is at the specimen midplane, magnitude of A considerably increases to the specimen free surface. The constraint parameter A at the specimen midplane diminishes when relative thickness t/W changes from 0.1 to 0.5 and has more or less stable value after that. Comparison of different specimens show that the center cracked plate specimen has highest values of the constraint parameter A for all crack depths. The compact tension specimen demonstrates lowest values of A that are even less than its small scale yielding value.

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References

[1] Cherepanov, G.P., The propagation of cracks in a continuous medium, J. Appl. Math. Mech., 31 (1967) 503-512.
[2] Rice, J.R., A path independent integral and the approximate analysis of strain concentration by notches and cracks, J. Appl. Mech. ASME, 35 (1968) 379-386.
[3] Hutchinson, J.W., Singular behavior at the end of a tensile crack in a hardening material, J. Mech. Phys. Solids, 16 (1968) 13-31.
[4] Rice, J.R., Rosengren, G.F., Plane Strain deformation near a crack tip in a power law hardening material, J. Mech. Phys. Solids, 16 (1968) 1-12.
[5] Betegon, C., Hancock, J.W., Two-parameter characterization of elastic-plastic crack-tip fields, J, Appl, Mech., 58 (1991) 104-110.
[6] O'Dowd, N.P., Shih, C.F., Family of crack-tip fields characterized by a triaxiality parameter. - I. Structure of fields, J. Mech. Phys. Solids, 39 (1991) 989-1015.
[7] Yang, S., Chao, Y.J., Sutton, M.A., Higher-order asymptotic fields in a power-law hardening material, Eng. Fract. Mech., 45 (1993) 1-20.
[8] Nikishkov, G.P., An algorithm and a computer program for the three-term asymptotic expansion of elastic–plastic crack tip stress and displacement fields, Eng. Fract. Mech. 50 (1995) 65-83.
[9] Nikishkov, G.P., Bruckner-Foit, A., Munz, D., Calculation of the second fracture parameter for finite cracked bodies using a three-term elastic-plastic asymptotic expansion, Eng. Fract. Mech., 52 (1995) 685-701.
[10] Delorenzi, H.G., Energy release rate calculations by the finite element methods, Eng. Fract. Mech., 21 (1985) 129-143.
[11] Li, F.Z., Shih, C.F., Needleman, A., A comparison of methods for calculating energy release rates, Eng. Fract. Mech., 21 (1985) 405-421.
[12] Nikishkov, G.P., Atluri, S.N., Calculation of fracture mechanics parameters for an arbitrary three-dimensional crack by the equivalent domain integral method, Int. J. Numer. Meth. Eng., 24 (1987) 1801-1821.
[13] Anderson, T.L., Fracture Mechanics: Fundamentals and Applications, CRC Press, Boca Raton, (2005).