



Elastic compliance of single-edge-notched tension SE(T) (or SENT) specimens

B. Tyson¹

CanmetMATERIALS, Hamilton, ON, Canada

btyson@nrcan.gc.ca

P. Ding, X. Wang

Carleton University, Ottawa, Canada

pingding@cmail.carleton.ca, xwang@mae.carleton.ca

ABSTRACT. There has been a trend recently to use specimen geometries for toughness measurement that are more representative of actual flaw geometries in service. A prominent example is the use of single-edge-notched tension specimens for assessment of surface flaws in pipelines. To obtain a resistance (R) curve, i.e. J-integral or CTOD as a function of crack growth, it is necessary to monitor the crack size as a function of J or CTOD. To facilitate obtaining these data from a single specimen, the elastic CMOD unloading compliance C has been used in several testing programs to estimate crack size. C is a function of several variables in addition to crack size – notably, specimen constraint (plane stress or plane strain). In this paper, the dependence of C on these variables will be discussed.

KEYWORDS. SENT; SE(T); Constraint; Compliance; R-curve.

INTRODUCTION: SE(T) TESTING

Characterization of resistance to fracture of the materials used in construction of engineering structures such as pipelines is a vital step in design. Conventionally, this has been done by using a test that gives a conservative material property (the “fracture toughness”) that can be compared with the estimated maximum crack driving force of a plausible flaw in service, and ensuring that the material selected has adequate toughness to prevent fracture. This is a safe way to proceed, but because of the high degree of conservatism in some cases it can lead to uneconomic design. In particular, for thin-walled structures such as pipe, the lower degree of constraint experienced by surface flaws can enable the material to withstand much higher driving force than in a highly-constrained test specimen. The logical response to this is to reproduce the actual service geometry and loads as closely as possible in the toughness test set-up. The resistance displayed by the material in this arrangement may not be a lower bound for the material, i.e. it may be “geometry-dependent”, but it will be the appropriate toughness to use in assessing the defect tolerance of the structure being simulated.

The procedure for engineering critical assessment of circumferential surface flaws (i.e. weld defects) in line pipe is a case in point. It is well known that the constraint for such flaws is substantially lower than the constraint in the standard three-point-bend test geometry. To generate toughness measurements more representative of the service conditions, tests with

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specimens of the same thickness as the pipe, notched in the same orientation as the surface flaws being assessed and loaded in tension to simulate service loads, have been developed. Several variants of the test, using single-edge-notched specimens loaded in tension (SENT or SE(I)), are in existence. The first, RP-F108 published by Det Norske Veritas [1], uses multiple specimens. The RP-F108 specimens are of preferred cross-section 2BxB (thickness x width: the dimension B here refers to *specimen width*, which is the *pipe wall thickness* in this case; normally, B refers to the *specimen thickness*, but there is no confusion for BxB cross-section specimens which will be the focus of the remainder of this paper), and “daylight” (distance between grips) of H=10B (i.e. 10W), tested in tension. In the years since RP-F108 appeared, many papers dealing with SENT testing have been published as well as two test methods in draft standard form, i.e. CANMET’s recommended procedure [2] and ExxonMobil’s procedure for measurement of CTOD using SENT specimens [3]. The latter two tests report single-specimen methods relying on a crack-mouth-opening (CMOD) unloading compliance (UC) technique to monitor crack size during the test. The intent of the present report is to discuss details of the UC method.

To estimate crack size using UC, a relationship between crack size and compliance is required. Estimation of this relationship is straightforward using standard linear-elastic finite element methods available in several software codes, and has been performed in many laboratories around the world. However, there are some subtleties in application of the results that should be recognized, in particular the relationship between plane strain and plane stress and which of these is the closest approximation to the actual test constraints.

ESTIMATION OF CRACK SIZE - PLAIN STRESS/STRAIN

The most straightforward procedure to obtain stress and displacement using finite element analysis (FEA) is to use a two-dimensional (2D) plane strain model. Plane stress constraints are difficult to simulate, and resort is normally made to a “generalized plane stress” formulation for this case. The resulting compliance data can be expressed in terms of a parameter u (sometimes called the “normalized compliance”):

$$u = 1 / (\sqrt{BCE'} + 1) \quad (1)$$

where B=specimen width, C=CMOD compliance (displacement/load), and E' is the “effective modulus”. For plane-strain compliance calculations, the appropriate “effective modulus” is the “plane-strain modulus” $E' = E / (1 - \nu^2)$ where ν is Poisson’s ratio for the material. The actual specimen constraints are, of course, not plane strain but rather something between plane strain and plane stress (see, for example, [4]). To obtain the plane stress compliance from Eq. (1), the “plane stress modulus” $E' = E$ should be used, where E is Young’s modulus for the material. In practice, the relationship between a/W and u is first found from plane-strain FEA calculations, and expressed in a convenient form, normally a polynomial for a/W as a function of u . Then, to estimate a/W from compliance, the value of u using the “effective modulus” appropriate to the constraint of the test specimen is calculated from the measured compliance. Then, a/W is calculated from the polynomial for a/W as a function of u .

Polynomial expressions for a/W have been published in a number of papers. It has become conventional to use a “daylight” H between grips of 10W and fixed-grip (clamped) load application, and most use a BxB cross-section geometry; all of the results discussed here are for this condition. All previously published results are in essential agreement. According to Shen et al. [2],

$$a/W = 2.044 - 15.732u + 73.238u^2 - 182.898u^3 + 175.653u^4 + 60.930u^5 - 113.997u^6 - 113.031u^7 + 8.548u^8 + 142.840u^9 \quad (2)$$

This expression is rather cumbersome, but since the equation is intended to be valid over the extended range of a/W between 0.05 and 0.95, to maintain accuracy it was found necessary to include ten terms in the polynomial. Another polynomial was published earlier by Cravero and Ruggieri [5]:

$$a/W = 1.6485 - 9.1005u + 33.025u^2 - 78.467u^3 + 97.344u^4 - 47.227u^5 \quad (3)$$

This equation contains only six terms, as it is valid only over the range $0.1 \leq a/W \leq 0.7$. This is the relevant range for practical testing, and in that range it agrees well with Eq. (2). For purposes of the test standard then, Eq. (3) is adequate. Fig. 1 shows a comparison of Cravero’s Eq. (3) with Shen’s FEA data [2] and with data calculated in the present study. Note that Cravero’s equation (shown as a solid curve in Fig. 1) provides an excellent fit to Shen’s and the present results over the range $0.1 \leq a/W \leq 0.7$, even up to $a/W=0.8$. There is a slight discrepancy for $a/W=0.05$ and $a/W=0.9$ and 0.95 ; however, these values of a/W are outside of the range expected to be used in practical tests. The conclusion is that Cravero’s equation is suitable for estimating crack size from CMOD compliance over the range $0.1 \leq a/W \leq 0.8$.



To demonstrate the influence of constraint, Fig. 2 shows the CMOD compliance (multiplied by the specimen thickness B ; E was taken as 207 GPa) calculated from Cravero's equation using plane strain and plane stress moduli. Also shown in the figure is the plane strain, stress (represented by one value at $a/W=0.6$) and 3D compliance calculated using FEA in the present study.

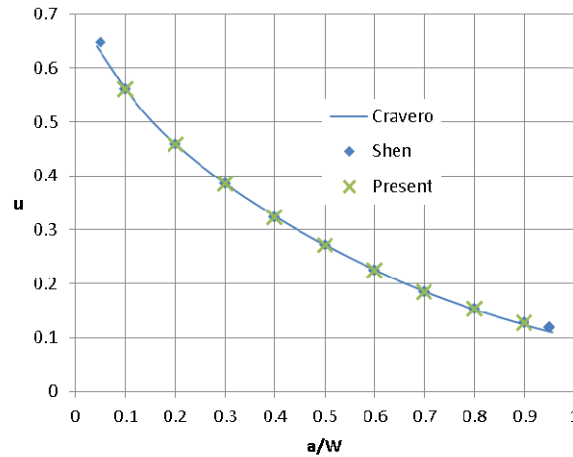


Figure 1: Normalized compliance as a function of a/W .

The 3D compliance is substantially larger than the plane strain compliance, and almost identical to the plane stress compliance. This is shown more clearly in the expanded portion of Fig. 2 shown in Fig. 3. Moreira and Donato [6] have confirmed for SE(I) specimens with $W/B=0.5, 1, \text{ and } 2$ and $a/W=0.1$ to 0.7 that the 3D compliance is correctly predicted by the plane stress model, although their results are for $H/W=6$ and so are not directly comparable to the data in this paper for $H/W=10$. Donato has also confirmed (private communication) that the FEA plane stress and plane strain compliances are the same function of u when u is normalized by the modulus $E'=E$ for plane stress and by $E'=E/(1-\nu^2)$ for plane strain.

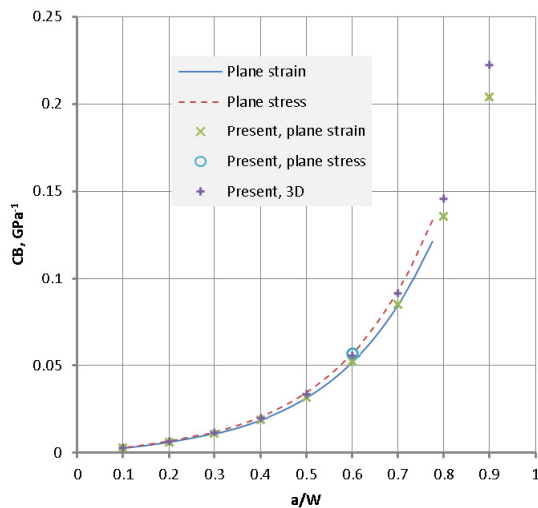


Figure 2: Variation of CMOD compliance with a/W .

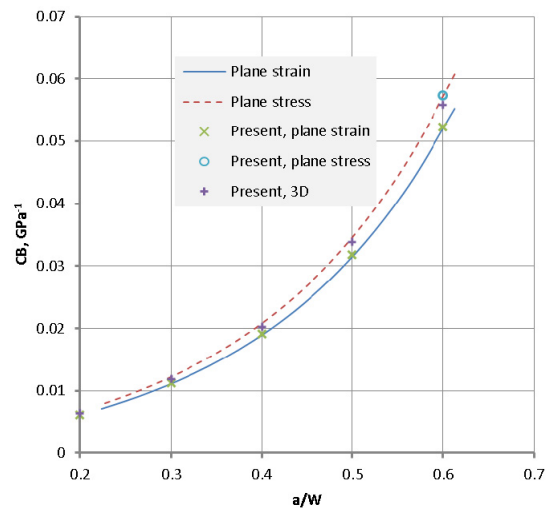


Figure 3: Expanded portion of Fig. 2.

VALIDATION

CANMET has recently completed a round robin designed to assess the viability of the single-specimen method developed earlier [2]. The results have been published recently [7]. Participants reported both calculated values (from unloading compliance) and measured values (from nine-point optical size measurements on the fracture surface) of the crack size. The results are shown in Fig. 4 and in Tab. 1.

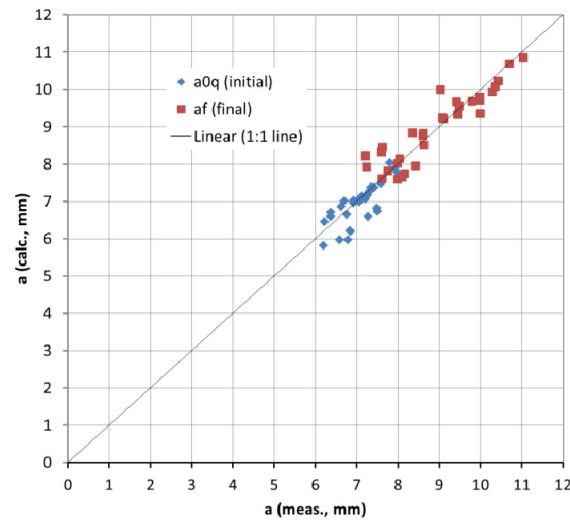


Figure 4: Comparison of calculated crack size with measured size.

Lab	Spec.	a_{0q} , mm		$\frac{a_{0q}}{a_0}$	a_f , mm		$\frac{a_{fcalc}}{a_{meas}}$	$\frac{(a_f - a_{0q})_{R-curve}}{(a_f - a_0)_{\theta-point}}$
		calc.	meas.		calc.	meas.		
A	HT-1	6.92	6.95	0.996	9.32	9.44	0.987	0.964
	HT-3	7.13	7.13	1.001	9.69	9.79	0.989	0.963
	HT-5	7.07	7.08	0.998	8.50	8.63	0.985	0.925
	HT-7	7.37	7.36	1.000	10.23	10.43	0.981	0.937
D	5108	6.93	7.02	0.987	9.35	9.99	0.936	0.960
	5208	6.63	6.85	0.967	9.72	9.99	0.973	0.933
	5407	7.80	8.04	0.970	9.94	10.29	0.966	0.896
	5409	6.77	6.65	1.017	9.67	9.41	1.028	1.004
E	RR5102	7.99	7.77	1.027	10.69	10.82	0.988	0.977
	RR5109	6.20	5.82	1.066	8.76	8.61	1.017	0.915
	RR5308	7.09	7.09	1.000	9.80	9.98	0.982	0.936
	RR5410	7.21	7.06	1.022	10.08	10.36	0.973	0.869
I	5103	7.50	6.81	1.101	8.83	8.62	1.025	0.646
	5205	7.50	6.73	1.114	9.99	9.03	1.106	0.841
	5406	7.28	6.59	1.104	8.84	8.35	1.059	0.840
J	5504	6.38	6.59	0.968	7.75	8.16	0.950	0.873
	5505	6.22	6.46	0.963	7.59	7.99	0.950	0.901
	5507	6.38	6.70	0.952	7.65	8.10	0.944	0.901
	5509	6.70	7.01	0.956	7.95	8.43	0.943	0.880
L	5401	7.34	7.36	0.997	9.55	9.48	1.007	1.025
	5104	7.27	7.20	1.009	9.22	9.11	1.012	1.027
	5101	7.91	7.88	1.004	10.85	11.04	0.983	0.928
	5508	7.30	7.25	1.007	9.24	9.07	1.019	1.061
N	5408	7.59	7.48	1.016	8.15	8.04	1.014	0.875
	5105	7.26	7.17	1.013	7.82	7.76	1.008	0.956
	5201	7.43	7.36	1.008	8.02	7.99	1.004	0.921
	5202	7.06	6.97	1.013	7.61	7.68	0.991	0.938
O	RRT01	6.849	6.184	1.108	8.32	7.61	1.094	1.035
	RRT02	6.842	6.213	1.101	8.45	7.62	1.109	1.142
	RRT03	6.784	5.974	1.136	8.22	7.21	1.140	1.161
	RRT04	6.582	5.963	1.104	7.92	7.24	1.094	1.125
Avg.				1.023		1.008	0.947	
Std. Devn.				0.053		0.053	0.100	

Table 1: Results of CANMET round robin: a_{0q} and a_0 are initial calculated and measured crack size values respectively, and a_f is the final crack size.



As shown in the last row of the table, the calculated and measured initial values agree on average within 2.3%, with a standard deviation of individual estimates of 5.3%. Calculated final values are accurate (i.e. agree with measured values) to within 0.08%, with a standard deviation of individual measurements of 5.3% (fortuitously identical to the standard deviation of initial crack size values). The estimate of crack growth, being the difference between final and initial crack sizes, is more sensitive to errors in size measurement; as shown in the last column, the calculated crack growth is accurate to 94.7% of measured values, with a standard deviation of individual measurements of 10%. It should be noted that a correction for rotation is required to use the UC method for a SE(T) specimen because the alignment of the specimen changes during crack growth. The specimen rotates so that the load line moves closer to the centre of the ligament, and the compliance changes accordingly. A correction factor derived by Shen and Tyson [8] is incorporated in the CANMET procedure [2] and was applied in the round robin. The correction is not large; for a deep crack ($a/W \approx 0.5$) the measured compliance is smaller by about 9% when the ligament is fully yielded than it would be for a straight specimen (to which the UC crack size equation applies), and the correction increases the calculated crack size by about 3%. Observation of specimens removed from the testing machine showed that yielding was confined to the ligament; the deduction is that rotation occurred by elastic deformation of the arms of the specimen.

CONCLUSIONS AND DISCUSSION

The principal conclusions of this study are:

- 1) for SE(T) specimens with $H=10W$ and BxB cross section, the parameters reported by Cravero and Ruggieri [5] represent well the relation between crack size and compliance; and
- 2) the effective modulus E' used in the normalized compliance u should be the “plane stress modulus” E .

Conclusion 2 is consistent with the approach in ASTM E1820-11e [9], in which E rather than $E/(1-\nu^2)$ is used in the UC equation for crack size as a function of u in for SE(B) specimens. At first glance it may seem strange that the plane stress modulus should be used for calculations in a test designed to measure the plane strain toughness. However, it is readily appreciated that the compliance is a function of displacements everywhere in the specimen and not just near the crack tip. The crack tip may well be in a state close to plane strain, especially if side grooves are used, and this justifies the use of the plane strain modulus in calculations relating the stress intensity factor K and the J-integral J , i.e. $J=K^2/E'$. However, the constraint for the bulk of the specimen, especially in tension-loaded specimens, is closer to plane stress than to plane strain.

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REFERENCES

- [1] Det Norske Veritas, Fracture control for pipeline installation methods introducing cyclic plastic strain, Recommended Practice DNV-RP-F108 (2006).
- [2] Shen, G., Gianetto, J. A., Tyson, W.R., Development of procedure for low-constraint toughness testing using a single-specimen technique, CANMET Report No. 2008-18(TR) (2008).
- [3] ExxonMobil, Measurement of crack-tip opening displacement(CTOD) fracture resistance curves using single-edge notched tension (SENT) specimens, ExxonMobil Upstream Research Company, Houston (2010).
- [4] Barber, J.R., *Elasticity*, Kluwer Academic Publishers, Dordrecht, The Netherlands (1992).
- [5] Cravero, S., Ruggieri, C., Estimation procedure of J-resistance curves for SE(T) fracture specimens using unloading compliance, Eng. Fracture Mech., 74 (2007) 2735-2757.



- [6] Moreira, F., Donato, G.H.B., Compliance solutions including 3-D and side-groove effects applicable to crack size estimation using C(T), SE(B) and clamped SE(T) specimens, In: 68th ABM International Congress, Belo Horizonte, MG, Brazil (2012).
- [7] Tyson, W.R., Gianetto, J.A., Low-constraint toughness testing: results of a round robin on a draft SE(T) test procedure, In: Proc. ASME 2013 Pressure Vessels & Piping Division Conference PVP2013, Paris, paper PVP2013-97299 (2013).
- [8] Shen, G., Tyson, W.R., Crack size evaluation using unloading compliance in single-specimen single-edge-notched tension fracture toughness testing, *Jour. of Testing and Evaluation*, ASTM, 37 (4) (2009) ID JTE102368.
- [9] ASTM E1820-11e, Standard Test Method for Measurement of Fracture Toughness, American Society for Testing and Materials, Conshohocken, PA (2011).