Can we describe kinetics of fatigue crack growth without influence of R-ratio?

M. Szata[1] G. Lesiuk[1]

[1] Wrocław University of Technology, Faculty of Mechanical Engineering, Smoluchowskiego 25, PL-50371 Wrocław, Poland.
mieczyslaw.szata@pwr.wroc.pl, grzegorz.lesiuk@pwr.wroc.pl.

ABSTRACT. *A new energy method of describing crack growth rate is proposed. The R-ratio is one of the main parameters which influence the experimental kinetic fatigue fracture diagrams da/dN - DK. The present-day methods of constructing kinetic fatigue fracture diagrams on the basis of energy dissipation in each loading cycle (related to a hysteresis loop area) make it possible to obtain a model invariable in relation to stress ratio. In this paper, the comparison of these two methods, their faults and features, as well as the results obtained for selected types of steel have been presented. For the experimental verification, the results of fatigue crack propagation studies for 18G2A and 40H steels have been utilized. In contradiction to the force factor Kmax, the energy parameter DH describes synonymously the fatigue crack propagation rate, independently on a stress ratio R. The linear dependence of crack propagation rate da/dN on energy dissipation of plastic deformation ahead of the crack tip for one loading cycle has been discussed with taking into consideration the consequences for fitting models in double logarithmic axes.*

INTRODUCTION TO DESCRIPTION OF FATIGUE CRACK GROWTH

The description of fatigue failure process, and the kinetic fatigue failure diagrams (KFFD), constitute a valuable tool in the engineering practice referring to the prediction of fatigue crack growth. The initial quantity is a fatigue crack growth rate expressed in [mm/cycle] or in [m/cycle], as a function of quantity drawn (most often) from fracture mechanics – the stress intensity factor, or the J integral. Mathematical description of experimental curves strongly depends on the stress ratio R, which has been shown in Fig. 1. Influence of the stress ratio is reflected in numerous empiric formulas describing a fatigue crack growth rate, e.g. in the Forman (1) or Walker (2) formula:

$$\frac{da}{dN} = \frac{C(\Delta K)^m}{(1-R)K_C - \Delta K} \tag{1}$$

$$\frac{da}{dN} = \frac{C_w}{(1-R)^{n_w}} (\Delta K)^{m_r} \qquad (2)$$

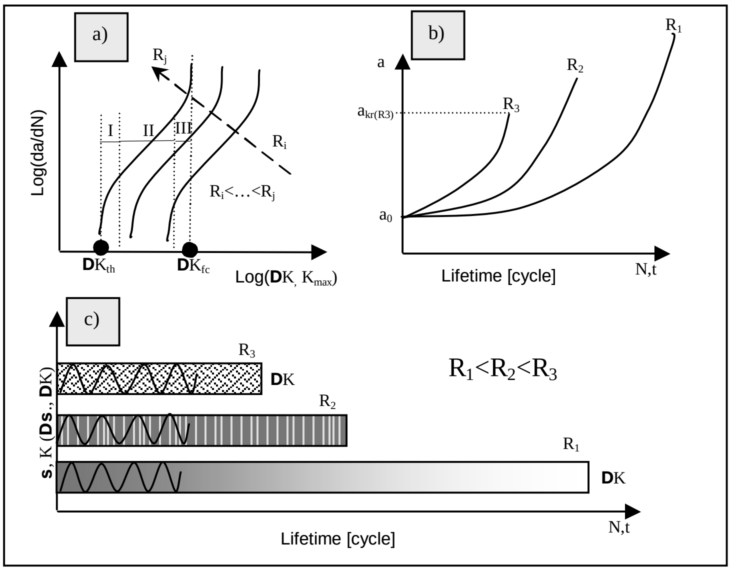

Fig. 1. Influence of stress ratio R on: a) the kinetics of fatigue failure performed on KFFD diagrams, b) crack extension against number of cycles, c) fatigue lifetime

The K_c quantity appearing in the formula (1) is a crack resistance for given load conditions. In the case of obstacles in its determining, it is often substituted with a static value K_{IC}. Exact determining of m and C constants requires knowledge of kinetic fatigue failure diagrams for different values of stress ratio. Instead, in the Walker formula, the C_w constant is determined experimentally for different ranges of R, and for R = 0 it corresponds to the Paris constant C. The quantity of $m_r = m + n$ designates a constant determined also by extrapolation of the data from KFFD. The stress ratio R essentially influences also the change in a threshold value of stress intensity factor ΔK_{th}, which can partially be observed in Fig. 1.a). Moreover, its influence strongly determines the analytic description of the process of closing the fatigue crack. A model involving the above problems is the Forman-Mettu model known in the subject bibliography:

$$\frac{da}{dN} = C\left[\left(\frac{1-f}{1-R}\right)\Delta K\right]^n \frac{\left(1-\frac{\Delta K_{th}}{\Delta K}\right)^p}{\left(1-\frac{K_{max}}{K_c}\right)^q} \qquad (3)$$

In the Forman-Mettu model (3) the c, n, p and q are experimental constants, f is a function of crack opening – a shape of that function could be determined based on European procedures FITNET [2]. Alternative methods of fatigue fracture kinetics description are also searched, which could eliminate the problem of stress ratio versus crack growth rate. As it has been shown by the results of study conducted by the authors, such a kind of KFFD could be obtained, explicitly describing fatigue crack growth rate, irrespectively of stress ratio *R*. However, for such effective structure of mathematical model, the other criterion than that of local force (based at K) has been used.

ENERGY APPROACH IN DESCRIPTION OF FATIGUE CRACK PROPAGATION

Energy, as a quantity combining the "force" and „displacement" measures seems to be naturally predestined for the description of fracture kinetics. Major part of hypotheses concerning both, fatigue and fatigue cracking description, is based on energy irrevocably dissipated in each cycle of a load spectrum. The dissipated energy accumulated in the material can be recorded during the tests in the form of a hysteresis loop. Determining the subcritical period of crack propagation using the energy method presented in [4,5,7] requires application of the first thermodynamics principle. Assuming the homogeneous disc subjected to sinusoidal loads (σ_{min}-σ_{max}) along with the central elyptic crack as a body model, the balance can be formulated as follows:

$$A + Q = W + K_e + \Gamma \tag{4}$$

In the equation, A quantity represents a work of external loads after N cycles, Q is a heat delivered to body during loading, W is the energy of strain after N cycles. Kinetic energy of the body has been marked as Ke, and Γ indicates the destruction energy related to increase in fatigue crack surface by ΔS. While formulating the energy balance (4) according to the work [5], the quantities A, Q, W, K_e, and Γ have been referred to the thickness unit. A slow propagation of crack during each cycle has also been assumed, thus the kinetic energy and heat exchanged within the process (for low frequencies of loading) can be neglected. After differentiating the equation (4) and after reductions we come to:

$$\frac{\partial A}{\partial N} = \frac{\partial W}{\partial N} + \frac{\partial \Gamma}{\partial N} \tag{5}$$

Szata [5] represents the Γ quantity as:

$$\Gamma = W_c + W_s \,, \tag{6}$$

where W_c designates the energy of cyclic plastic strains and, by analogy, W_s represents a static (monotonic) component of the energy, corresponding to $\sigma_{zw\max}$. Γ has been defined as a maximum value of dissipation of a static energy component on plastic strain (Γ=W_{smax}), which initiates a proces of fracture without participation of energy for cyclic strain changes (W_c=0) [5]. After elementary transformation (5) and considering (6) the general form of the crack surface propagation the equation is as below:

$$\frac{\partial S}{\partial N} = \frac{\partial W_c / \partial N}{\partial(\Gamma - W_s)/\partial S} \tag{7}$$

In order to obtain the expression based on which the experimental diagram of fatigue fracture kinetics is to be created, the simplified forms of (8) and (9) are to be used, without considering the change in the function of length in crack opening δ:

$$\frac{\partial \Gamma}{\partial S} = \sigma_{plf}\varepsilon_{fc} \tag{8}$$

$$\frac{\partial W_S}{\partial S} = \sigma_{plf}\varepsilon_{f\max} \tag{9}$$

Determining the exact forms of (8) and (9) has been presented in the work [5], based on the Dugdale – Panasiuk model. Considering the (8) and (9), the expression being the denominator of (7) can be obtained:

$$\frac{\partial(\Gamma - W_S)}{\partial S} = s_{plf}e_{fc} - s_{plf}e_{f\max} = s_{plf}e_{fc}(1 - \frac{e_{f\max}}{e_{fc}}) \tag{10}$$

The equation (10) can also be presented somewhat differently:

$$\frac{\partial(\Gamma - W_S)}{\partial S} = s_{plf}e_{fc} - s_{plf}e_{f\max} = s_{plf}e_{fc}(1 - \frac{e_{f\max}}{e_{fc}}) = s_{plf}e_{fc}(1 - \frac{K^2_{\mathrm{I}\max}}{K^2_{fc}}) \tag{11}$$

By designating $W_c^{(1)} = \partial W_c / \partial N$ as a quantity of plastic strain energy dissipation ahead of the crack tip for one load cycle we obtain:

$$\mathrm{dS}/\mathrm{d}N = \frac{W_c^{(1)}}{s_{plf}e_{fc}(1 - K^2_{\mathrm{I}\max}/K^2_{fc})} \tag{12}$$

The equation of fatigue crack growth rate assumes the form of:

$$\frac{da}{dN} = \frac{a\Delta H}{s_{plf}e_{fc}} \tag{13}$$

In the equation (13) α is a constant dimensionless coefficient, and ΔH is a new energy parameter depending on $W_c^{(1)}$, B is thickness of specimen, equal to:

$$\Delta H = W_c^{(1)} / B(1 - K_{\mathrm{I}\max}^2 / K_{fc}^2) \qquad (14)$$

Experimental validations

In order to verify the usefulness of the energy parameter ΔH for representing the fatigue failure process kinetics, an experiment was conducted in which two types of fatigue failure kinetics diagrams were recorded. The tests were performed for compact samples made according to Standard [1]. Also, the bar samples with a side stress concentrator of the 12x18x180 [mm] dimensions were tested. The bar samples were fixed in the support manner and subjected to bending with 1 [Hz] frequency, and the compact samples were streched. In course of the tests, the force F [N] and displacement [mm] along the force axis were recorded.

The measurement system was built of the following components:

- Hydraulic Pulsator MTS-810,
- Extensometer MTS (measuring base 2,5±0,5 [mm]),
- Optical system enabling objective movement in vertical and horizontal direction, as well as rotation around the vertical axis (reading accuracy 0,01 [mm]),
- PC with software supporting experimental tests (here the HP VEE) [7].

As the test materials two steel grades: 18G2A (0,2%C; 0,26%Mo; 0,2%Cu; 1,3%Mn; 0,03%S, 0,02%P) and 40H (0,4%C; 0,7%Mn; 1,1%Cr; 0,3%Si; 0,3%Ni; 0,03%S; 0,02%P), were used. Basic mechanical properties of the materials have been presented in Table 1.

Table 1. Strength properties of the tested steels

Material	R_m [MPa]	$R_e/R_{0,2}$[MPa]	A_5[%]	K_{fc}[MPa*m^{0,5}]
18G2A	600	350	22	105
40H	980	780	10	45, 80, 100*

The values of critical stress intensity factors (in table 1) K_{fc} are 45, 80, 100 [MPa*m^{0,5}], for the temperature 200, 450, 700 [°C], respectively. After obtaining the initial fatigue crack the samples were subjected to cyclic loads with step increase in force F, while maintaining the stress ratio R. The load level was increased by about 10% with frequency of 10 [Hz].

In course of the experiment, two types of fatigue fracture kinetics have been recorded. In the first one, the range of changes in the stress intensity factor ΔK appears, which characterises the intensity of cyclic strain in the crack tip, and in the second one, ΔH quantity appears, corresponding to the searched dissipation energy of strain recorded in the form of a hysteresis loop.

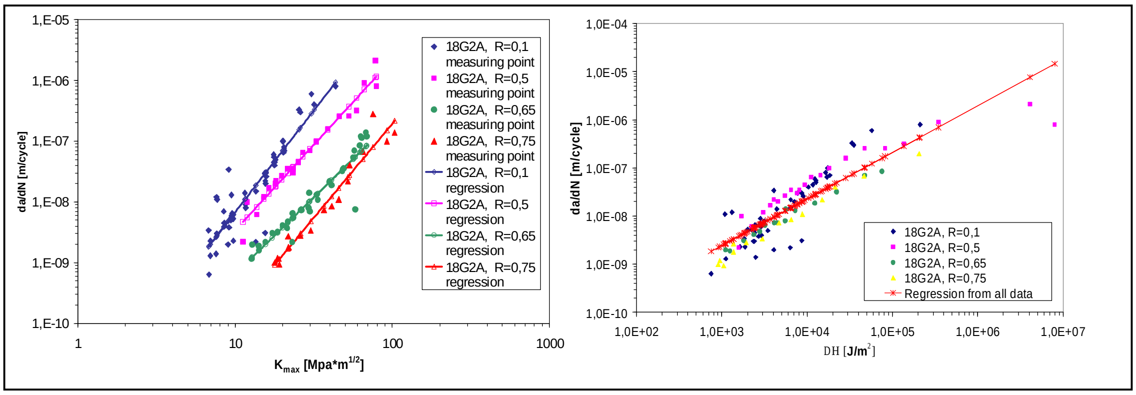

Fig. 2. Test results for the 18G2A steel

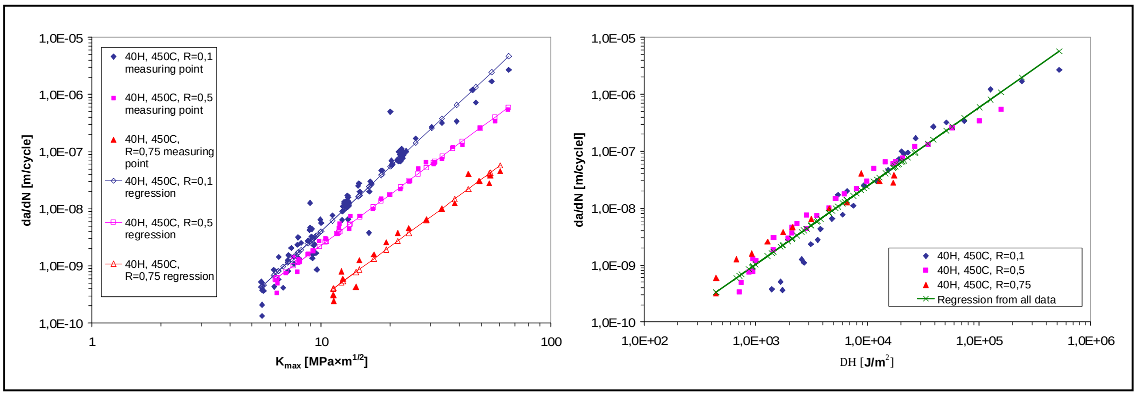

Fig. 3. Test results for the 40H steel (heat treatment 450ºC)

Comparison of theoretical and approximation formula – a method of equivalent surfaces

The closed solution of the kinetic equation (12) for any shape of a crack contour L seems to be difficult in the case, if we would like to use an accurate form of the equations (8) and (9). In order to simplify this solution the equivalent surface method has been proposed. The basis of this method is the hypothesis that the flat cracks occurring in the homogeneous field of tensile stresses with convex contours L and the identical surface areas S have the similar magnitudes of areas ΔS before a crack tip and the similar energies $W_s^{(f)}$ and $W_c^{(f)}$. Thus we can take as a representative a circle of the radius R and the area $S = \pi R^2$. Then, the equation of a fracture crack growth in the shape of a circle can be expressed as:

$$\frac{\partial S}{\partial N} = 0{,}0899 a\, s_{plf}^{-2} S\sqrt{S}\, s_{zw}^{4}(1-R)^{4}\left[K_{fc}^{2} - 0{,}9959 s_{zw}^{2}\sqrt{S}\right]^{-1} \qquad (15)$$

If we integrate the expression (15) at assumed final conditions $N=N_{cr}$, $S=S_{cr}$, denoting the number of cycles to destruction as Ncr, a critical surface as Scr, and initial conditions as N=0, $S=S_{in}$ (initial surface of a crack), then we obtain:

$$N_{cr} = 16{,}051 a^{-1} s_{plf}^{2} s_{zw}^{-2}(1-R)^{-4}\left[\sqrt{\frac{S_{cr}}{S_{in}}} - 1 - 0{,}5\ln\frac{S_{cr}}{S_{in}}\right]^{-1} \qquad (16)$$

In this case, the calculations will be considerably simplified. On the ground of this assumption, the kinetics of surface area changes for any crack with a convex contour *L* will be analogous to the kinetics of a circular crack with the same surface *S*. For example, let us consider an elliptic crack. Considering the volume limitations for the present work, the exact equivalents of the equation (15) and (16) for an ellipse have not been presented here. Based on the equivalent surface method (see Fig. 4), an almost 95% conformity of the exact and approximated by equivalent surface solutions have been achieved for the ellipse semi-axes ratio (x) greater than 0,3.

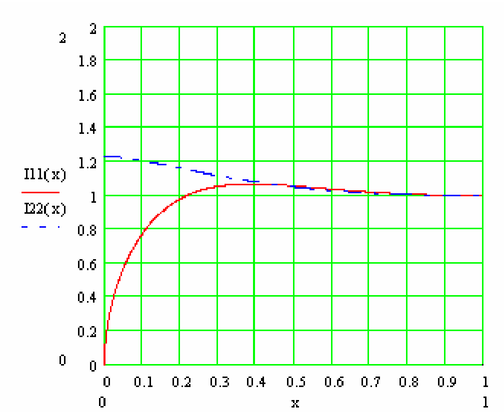

Fig. 4. Comparison of results of
the circular and elliptical crack [5]

The line I11(x) indicates a ratio of dissipated energy (of plastic strains ahead of the crack tip per one load cycle) determined in the exact way to the same quantity determined using the equivalent surface method. The line I22(x) instead, expresses the ratio of surface areas of a plastic zone ahead of the crack tip (a quotient of the exact and approximated solution). The results obtained using the equivalent surface method indicate for its usefulness in estimating both, the lifetime of a fatigue crack and the size of plastic zones, or the dissipated energy for cracks of any-shape smooth outline.

RESULT DISCUSSION AND CONCLUSIONS

On the contrary to the da/dN – K_{max} diagrams, at the da/dN – ΔH diagrams constructed for a given range of crack growth rate, the differences in fracture kinetics have not been observed. This indicates that, as opposed to the force parameter K_{max}, the energy parameter ΔH univocally defines the fatigue crack growth rate independently of the stress ratio R. Invariance of the diagram in relation to R constitutes some progress in the coherent description of the fatigue fracture kinetics.

Aplication of the energy criterion for fatigue crack growth rate is also an excellent tool for the parts subjected to multiaxis cyclic load. Modification of the formula (expanded for multiaxial state) describing the dissipated energy may be found in the work [4]. The equally interesting fact related to energy approach is the existence of an exponential relation with index of 4 for the stress intensity factor (K^4) in the exact solution to the equation (12) (it may be found in the work [5]). Other bibliography sources both, the older ones [9], as well as the contemporary ones [8], show that similar proportionality takes place while deriving the fatigue crack growth rate equation in the energy method. Should there be any regularity then?

New research possibilities (presented among others in the work [3]) and related to recording a hysteresis loop for ferromagnetic materials using the magnetic-mechanical phenomena (Villari effect), promise a coherent description of a fatigue process at the energy ground.

REFERENCES

1. ASTM STANDARDS E 399-1, *Standard Test Metod for Plane Strain Fracture Toughness of Metallic Material*
2. FITNET, *European Fitness for service Network*, 2006
3. Kaleta J., *Doświadczalne podstawy formułowania energetycznych hipotez zmęczeniowych*, Oficyna Wydawnicza Politechniki Wrocławskiej, Wrocław 1998..
4. Schlyannikov V. N., *Elastic-plastic mixed-mode fracture criteria and parameters*, Springer Verlag Berlin Heidelberg New York, Germany 2003
5. Szata M., *Opis rozwoju zmęczeniowego pękania w ujęciu energetycznym,* Oficyna Wydawnicza Politechniki Wrocławskiej, Wrocław 2002.
6. Szata M., Lesiuk G., *Algorithms for fatigue crack estimation using energy method*, Archives of Civil and Mechanical Engineering, 2009 (in press).
7. VEE HP, *HP VEE Visual Programming Language*, 1999
8. Yao Y., Fine M.E., Keer L.M.: *An energy approach to predict fatigue crack propagation in metals and alloys*. Int. J. Fract. (2007) 146: 149-158.
9. Raju K.N., *An energy balance criterion for crack growth under fatigue loading from considerations of energy of plastic deformation*, Int. J. Fract. (1972) Vol.8 No. 1.