In-Plane and Out-of-Plane Crack-Tip Constraint Effects under Biaxial Loading in Plastic and Creeping Materials

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ABSTRACT. *In-plane and out-of-plane constraint effects on crack tip stress fields under both elastic-plastic and creep conditions are studied by means of three-dimensional numerical analyses of finite size boundary layer models and plane strain reference solutions. Characterization of constraint effects in terms of the non-singular T-stress, the local triaxiality parameter h and Tz-factor is investigated. The influence of load biaxiality, mode mixity and creep time on the behavior of constraint factors are considered. The computational data for constraint factors variation under the plane strain are compared with the constraint parameter distributions for the finite thickness plate. It is found that under biaxial mixed mode loading the difference between the full three-dimensional elastic-plastic and creep stress fields and the plane strain reference solutions appears to depend on the distance to the crack tip and to the free surface of the plate.*

INTRODUCTION

Constraint effects in modern fracture mechanics is usually considered as specimen configuration and loading conditions influence on crack-tip fields. Therefore, fracture toughness dependence is referred to these factors and can't be used as constant of material. However, the general discussion of constrain effects required to be defined more exactly. Constraint effects can be defined as specimen prevention from plastic strains depended on geometry and loading conditions.

Constraint effects near the crack tip have long been extensively studied. Most of researches are referred to in-plane constraint. Since Williams [1] presented the asymptotic expansion of the stress-field around the crack-tip in elastic body that includes a non-singular in-plane normal stress component, the T-stress. Subsequently Larsson and Carlsson [2], Rice [3] have shown that including the *T*-stress gave an improved representation of the elastic-plastic crack tip stress fields. Based on T-stress a two-parameter approach J-T was proposed by Betegon and Hancock [4], which takes into account the in-plane constraint on crack-tip fields. O'Dowd and Shih [5,6] have introduced as alternative constraint methodology a two-parametrical approach on the base of J and a hydrostatic stress parameter Q. The $J - A_2$ two-parameter three-term approach was also proposed to describe the stress field in the vicinity of the crack tip in a power hardening material [7]. However, all these approaches can successfully describe the in-plane constraint effects, but they are limited to a planar case. The description of out-of-plane constraint should include specimen's dimension such as thickness. Only several researches have been done to describe thickness effect on the crack-tip fields under mixed mode loading [8-11].

In this paper, the remote boundary conditions were based on the first two terms of the Williams expansion of the elastic plane stress field. Different degree of mode mixity and T-stress are given by combinations of far-field stress level, biaxial stress ratio and initial crack angle. Full-field finite element analysis based on a modified layer approach wherein the elastic K-field as well as the T-stress is prescribed as remote boundary conditions is employed to model the effects of biaxial loading on nonlinear behavior under plane strain conditions. The geometry considered in detailed three-dimensional finite element calculations is biaxially loaded finite thickness plate. Loadings and initial crack angle were applied related to a range of far-field biaxial stress ratio (-1, +1) and mode mixity I/II (0, 1).

DETERMINATION OF OUT-OF PLANE CONSTRAINT FACTORS

It is well known that different traditional approaches, which successfully describe the in-plane constrain are not accurate for 3D cracks. Thus, it is necessary to use others factors to describe the out-of-plane constraint. The T_z factor introduced by Guo [8] is an important parameter to characterize the constraint effect accurately, which is essential to establish a three parameter dominated stress field and offers a possibility to characterize the stress-state in 3D cracked body

$$T_z = \frac{\sigma_{zz}}{\nu\left(\sigma_{xx} + \sigma_{yy}\right)}, \tag{1}$$

where ν is the Poisson's ratio, $\sigma_{xx}, \sigma_{yy}, \sigma_{zz}$ are the stress tensor components.

The triaxiality parameter h has been proposed by authors [12] because of the dependency of crack tip constraint and stress triaxiality:

$$h(r,\theta,z) = \sigma_{kk} \Big/ \left(3\sqrt{\frac{3}{2} s_{ij} s_{ij}}\right). \tag{2}$$

where σ_{kk} and s_{ij} are hydrostatic and deviatiric stresses, respectively. Being the quotient of the first invariant of the stress tensor and the second invariant of the stress deviator, it is heuristic local measure of energy available for material degradation and damage. As it expected the T_z- factor and triaxiality parameter h can be describe both in-plane and out-of-plane constraint effects.

The different combinations of load biaxiality and mode mixity are characterized by the in-plane elastic nonsingular term [13]

$$T = \left(\sigma/\sigma_y\right)\left(1-\eta\right)\cos 2\alpha \tag{3}$$

where η is nominal stresses biaxial ratio, σ_y is the yield stress, α is inclined crack angle. All radial distributions of both constraint factors are represented with respect to normalized crack tip distance. It then follows purely from dimensional considerations that for elastic-plastic problem the distance from the crack tip r must scale by the yield stress σ_y when the loading is governed solely by *J*-integral

$$\bar{r} = (\sigma_y r/J) = (r/a)(\sigma_y E/\sigma^2 \pi) \tag{4}$$

where σ is applied nominal stress and *a* is crack length. In the case of creeping material the loading governing parameter is the *C*-integral and a dimensionless polar radius is

$$\bar{r} = (\sigma_0 \dot{\varepsilon}_0 r/C) = (r/a)/\left(\pi\sqrt{n}\left(\frac{\sqrt{3}}{2}\frac{\sigma}{\sigma_0}\right)^{n+1}\right) \tag{5}$$

where σ_0 is a reference stress, $\dot{\varepsilon}_0$ is a reference creep strain rate and *n* is the creep exponent. Note that for creeping material, the amplitude factors depend generally on the creep time, magnitude of the applied loading, crack geometry and material properties. Due to the complexities in the present work considered different creeping stages. In this case it is usefull to normalize a current creep time *t* by the characteristic time t_T for transition from small-scale creep to extensive creep which is given as

$$\frac{t}{t_T} = \frac{t(n+1)EC}{(1-\nu^2)K_I^2} \tag{6}$$

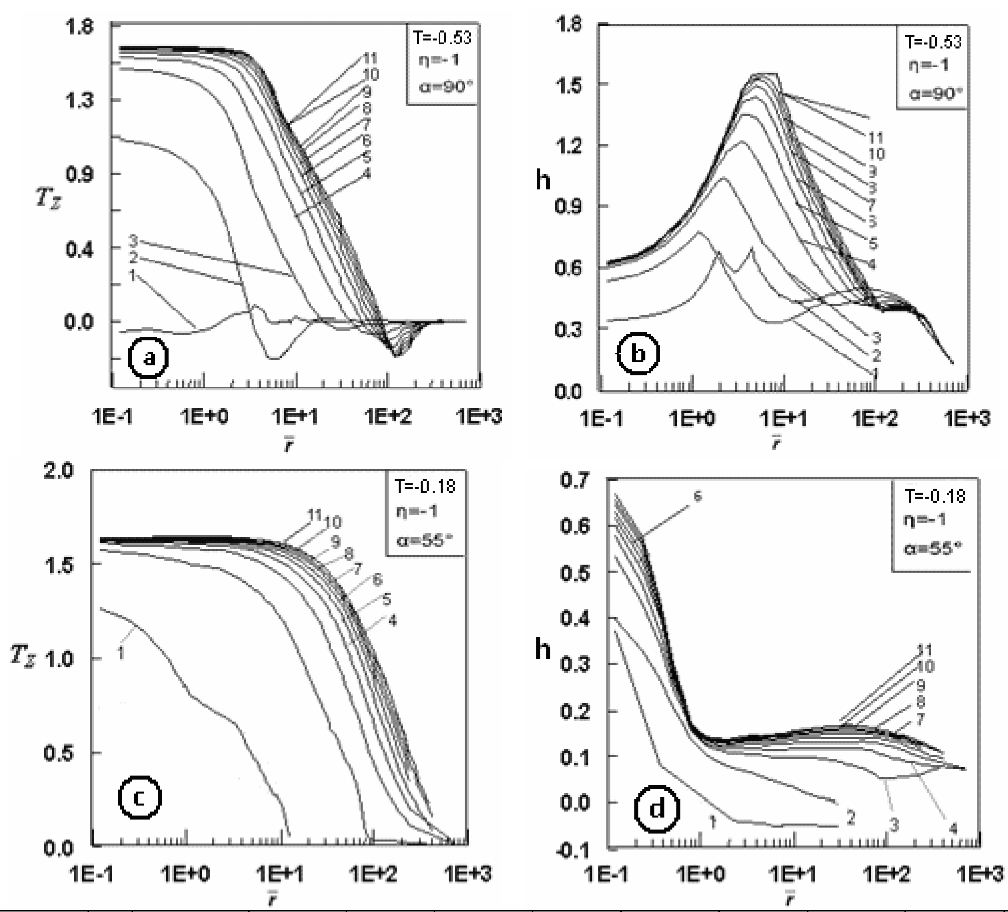
where K_I is elastic stress intensity factor, ν is the Poisson's ratio, *E* is the Young's modulus.

RESULTS AND DISCUSSION

Full-field 3D finite element analysis are carried out to determine the elastic-plastic and creep stress fields along the through-thickness inclined crack front in a circular disk subjected to biaxial mixed mode loadings. Different degree of mode mixity and T-stress are given by combinations of far-field stress level, biaxial stress ratio and initial crack angle. Loadings and initial crack angle were applied related to a range of far-field biaxial stress ratio (-1, +1) and mode mixity I/II (0, 1).

For FE model only a half of the circular disk is used because of the symmetry about the z-axis. The thickness of the layer in z-direction is gradually reduced toward the free surface (*z/b*=0) to accommodate the strong variations of the stress gradients through the half-thickness of the plate. In the mid-plane of the circular disk the dimensionless z-coordinate is *z/b*=0.5 where *b* is the plate thickness.

The 3D distributions of the constraint factors for given combinations of load biaxiality and mode mixity have been obtained by the 3D-FEM analysis for the material deformation behavior described by additive decomposition of the elastic, plastic and creep strains. For the present study, the Young's modulus, Poisson's ratio and the yield stress were considered to be 205 GPa, 0.3 and 380 MPa. The strain hardening exponent was 4.96. The creep parameter and the creep exponent are B=$1.4 \cdot 10^{-10}$ and n=3.

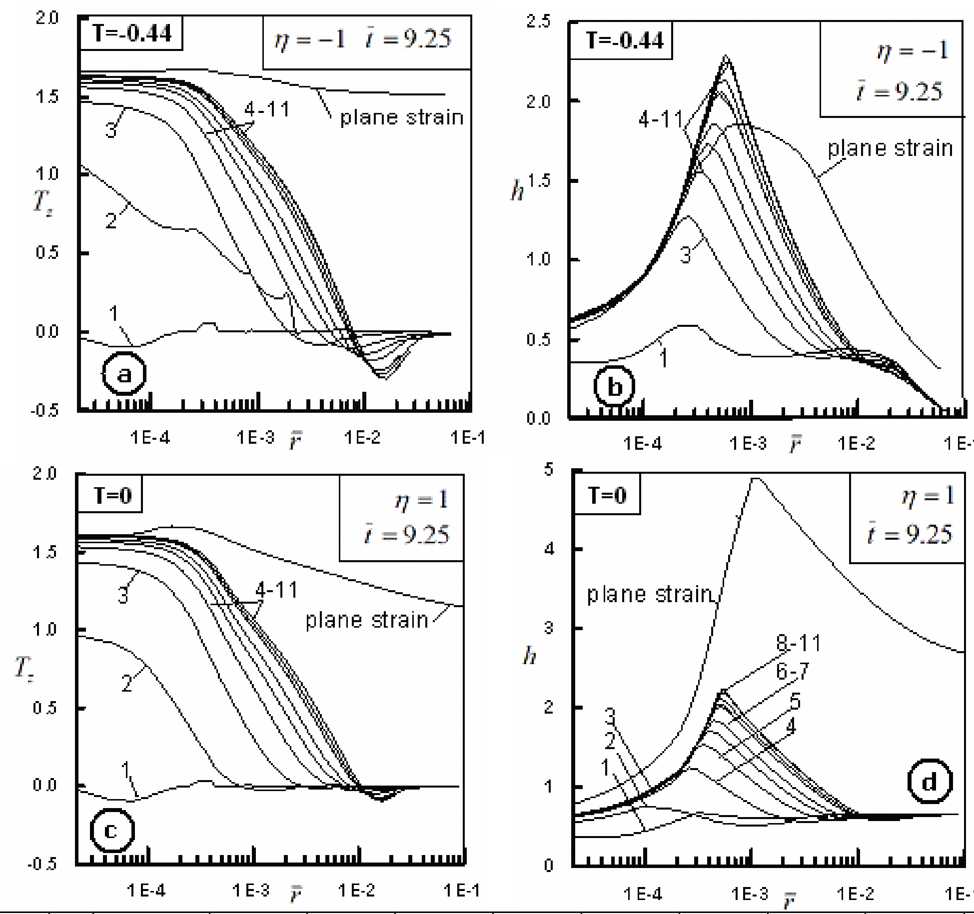


line number	1	2	3	4	5	6	7	8	9	10	11
thickness z/b	*0*	*0.0125*	*0.05*	*0.1*	*0.15*	*0.2*	*0.25*	*0.3*	*0.35*	*0.45*	*0.5*

Figure 1. Out-of-plane elastic-plastic constraint factors radial distributions

In Fig.1 the elastic-plastic radial distributions of the out-of-plane constraint factors along the crack front in the thickness direction are plotted for different inclination angles, i.e. mode mixity under equi-biaxial tension-compression of $\eta = -1$. Figs.1(a-b) depicts the behavior T_z-factor and stress triaxiality parameter h under pure mode I characterized by in-plane T-stress value of T=-0.53, while Figs.1(c-d) displayed mixed mode conditions at T=-0.18. As can be seen from these figures, the out-of-plane constraint factors decreases along the crack front toward the plate free surface when inclination angle altered from pure mode I to mixed mode with increasing crack tip distance.

Displayed in Fig.2 are the variations of the out-of-plane constraint factors radial distributions under equi-biaxial tension-compression ($\eta = -1$) and equi-biaxial tension ($\eta = +1$) loading characterized by in-plane constraint parameters of T = -0.44 and T = 0,



line number	1	2	3	4	5	6	7	8	9	10	11
thickness z/b	*0*	*0.0125*	*0.05*	*0.1*	*0.15*	*0.2*	*0.25*	*0.3*	*0.35*	*0.45*	*0.5*

Figure 2. Out-of-plane creep constraint factors radial distributions

respectively. These distributions related to extensive creep conditions with dimensionless creep time $\bar{t}=9.25$. The corresponding constraint factors distributions for the reference plane strain problem are plotted for comparison purposes in Fig.2(a-d). It is found that under biaxial pure mode I loading the difference between the full three-dimensional creep stress fields and the plane strain reference solutions appear to depend on the distance to the crack tip and to the free surface of the plate. Moreover, the plane strain solution for the out-of-plane constaint T_z-factor and stress triaxiality parameter h coincide with the 3D finite size solids distribution only in the mid-plane (half-thickness of plate z/b=0.5) at short distance close to the crack tip. Results for the elastic-plastic materials the constraint parameters distributions under biaxial loading at pure mode I as a function of the crack front distance show similar trends.

Figs. 3-4 show the typical behavior of the out-of-plane constraint T_z-factor and the stress triaxiality parameter h along plate thickness direction for both plastic and

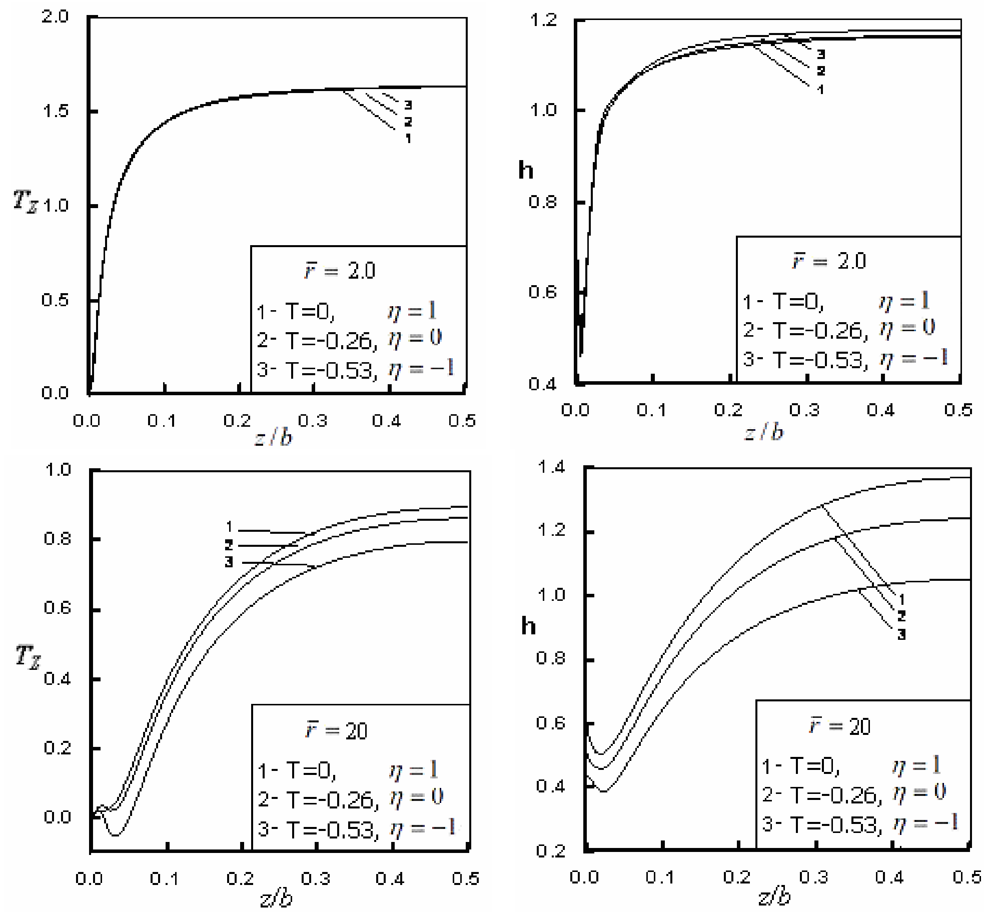


Figure 3. The T_z and h distributions along plate thickness direction for a plastic material

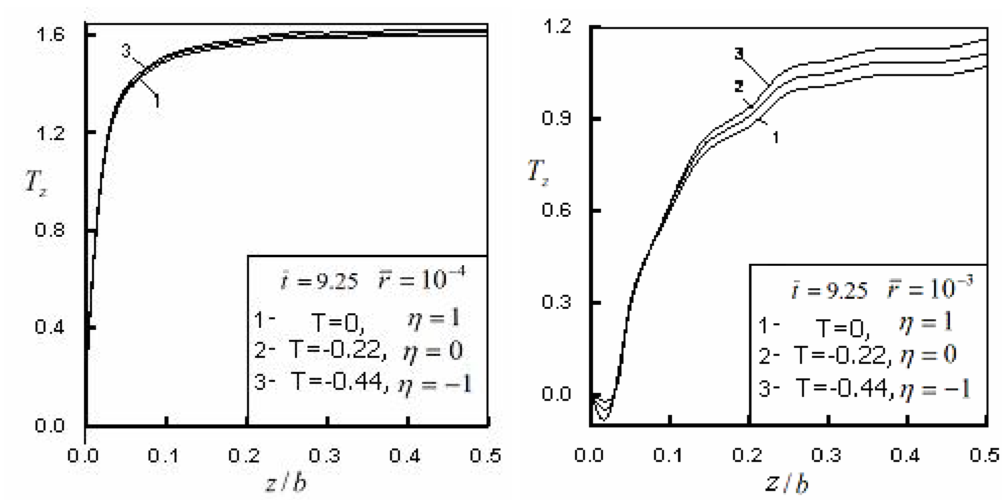
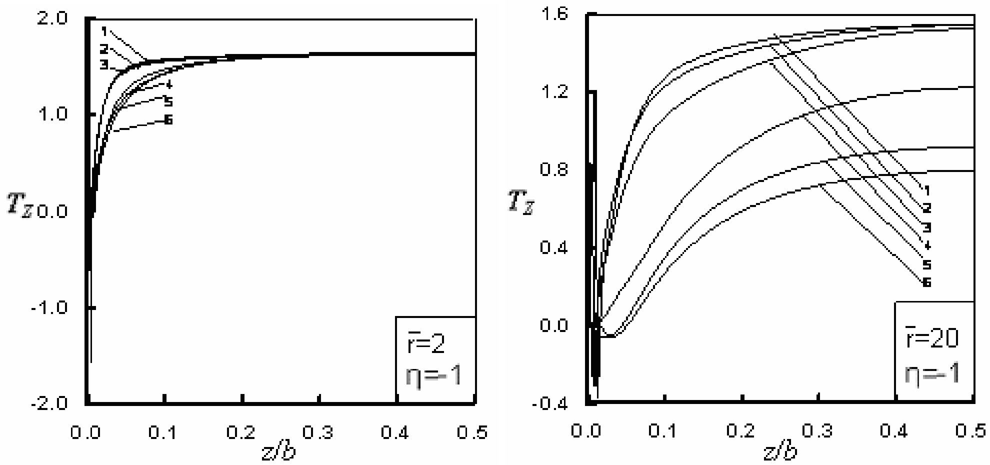


Figure 4. The T_z distributions along plate thickness for a creeping material

creeping materials under different biaxial pure mode I loading as a function of a given crack tip distance. As it follows from these figures for the some distance behind of unloading zone, the out-of-plane constraint factors are a monotonic decreasing function of the in-plane constraint factor *T*-stress, but is nearly independent of the load biaxiality at short distance close to the crack tip. It should be noted that the represented numerical solutions are accounting for border effect near the free surface of plate.



line number	1	2	3	4	5	6
α	55^0	65^0	70^0	80^0	85^0	90^0
T	-0,18	-0,34	-0,4	-0,49	-0,52	-0,53

Figure 5. The T_z distributions as a function of mode mixity and crack tip distance

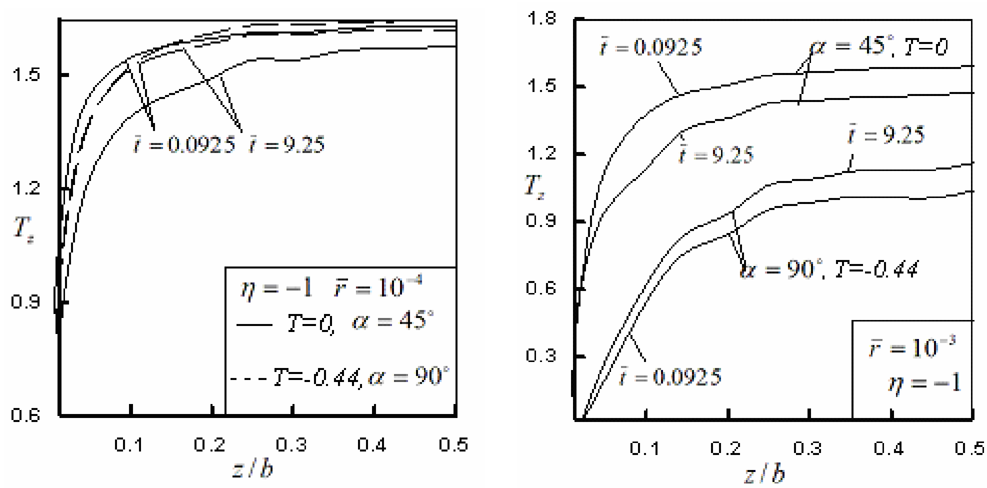


Figure 6. The T_z distributions as a function of creep time and mode mixity

Fig.5 illustrated the out-of-plane constraint factor distributions along plate thickness direction for elastic-plastic material under mixed mode biaxial loading of $\eta = -1$. It is found that at the distance close to the crack front $(\bar{r} = 2)$ mode mixity has no contribution to T_z which is nearly independent of in-plane *T*-stress. However, for a given crack tip distance equals the value $\bar{r} = 20$ the T_z-factor decreases gradually with decreasing in-plane *T*-stress or changing of crack angle α from mode II $\left(\alpha = 45^{\circ}\right)$ to mode I $\left(\alpha = 90^{\circ}\right)$ crack. The out-of-plane constraint factor T_z for mixed mode is dependent not only on *T*-stress, but also on distance to the free surface of finite thickness plate. As it follows from Fig.6 for creeping material T_z-factor is more depending on mode mixity than creep time. Generally, comparing of Figs.3-6 it can be concluded that the relations between the out-of-plane and the in-plane constraint factors characterized by T_z, *h* and *T*, respectively, show that the distributions along plate thickness are more sensitive to mode mixity than load biaxiality.

CONCLUSIONS

Two constraint concepts are analyzed with respect to their capability of characterizing load biaxiality and mode mixity effects. For a given radial distance the out-of-plane constraint factor T_z and the stress triaxiality parameter *h* are monotonic decreasing functions of the in-plane non-singular term *T* combining the influence of load biaxiality and mode mixity. It is found that the out-of-plane constraint factors behavior for the reference plane strain solution and the finite size 3D solids differ significantly with increasing of the crack front distance. Discrepancies in constraint parameters distributions along crack front towards the plate thickness have been observed under different biaxial loading conditions and creep time of the power law hardening material.

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