Plastic factor of front face compact tension specimens

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Abstract To estimate fracture toughness of irradiated material in nuclear engineering, the testing specimens with half thickness of a standard compact tension specimen are used to get the *J* resistance curve of materials according to Standard Test Method for Measured of Fracture Toughness (ASTM E1820). The normalization method recommended by ASTM E1820 is not directly applied in obtaining the fracture toughness of metallic materials because it does not provide the plastic factor of Front Face Compact Tension specimen with 12.5mm thickness and 25mm width (1/2 FFCT specimen). Based on the energy of *J* integral and the detailed finite element analysis, the plastic factor of a 1/2 FFCT specimen is presented. With the plastic factor of 1/2 FFCT, the *J* resistance curve of a 1/2 FFCT specimen and its fracture toughness can be obtained by Using the normalization method.

Keywords Front face compact tension, Load line compact tension, Plastic factor, Finite element, normalization method

1. Introduction

Structural materials, A508-Ⅲ steel which long-term service in the radiation environment , must have material damage and material aging in nuclear engineering. Long-term, department of design and operations has focused on the nuclear reactor material damage by surveillance and prevention. The problem that measuring the fracture behavior of irradiated material A508-Ⅲ steel is need to be solved as quickly as possible. The fracture behaviors of material include the fracture toughness and crack growth rate of material and so on. Based on single specimen method, the 1/2FFCT specimen is selected to estimate the fracture toughness. Because the test material is irradiated, all operations are completed by the mechanical arm. The unload compliance method require the high of neutral and test accuracy, so applying of unload compliance is restricted in fracture mechanics automatic test technology. The unload compliance method does not good coordinate with the mechanics arm. It is recommended that the normalization data reduction technology be used to in ASTM E1820-11 to determine *J* resistance curve and reduced test costs as well. The normalization data reduction technology involves the plastic factor of test bending specimen, so the plastic factor of 1/2FFCT specimen is quickly studied.

There are two ways to obtain the plastic factor which are SLF (Slip Line Field) theory [1-4] and FEA (Finite Element Analysis) [5-10]. The SLF theory assume that material behavior is perfectly elastic-plastic. It establishes the contact the plastic factor with the limit load. The FEA studies the plastic factor which considering the different stain hardening, so the FEA can more accurate analysis the plastic factor. Based on the *J* integral energy theory [11] and the detailed finite element, the plastic factor of compact tension specimen is researched.

2. Plastic Factor

2.1. Plastic Factor of Load Line Compact Tension Specimen

Rice [11] proposed the parameter *J*-integral for nonlinear elastic material as a measure of crack-tip singularity intensity of HRR field [12-13]. Begley and Landes [14-15] first recognized that the *J* integral and its critical value can be evaluated experimentally from the interpretation of *J* as the energy release rate. A method for estimating the *J* from a single load-displacement record was proposed first by Rice et al. [16]. For a bending specimen with different sized cracks, Sumpter and Tuner [17] proposed a general expression of *J* integral. A total Δ can be separated into an elastic component Δ_{el} and a plastic component Δ_{pl}, so the *J* integral would be expressed as

$$J = J_{el} + J_{pl} \tag{1}$$

where J_{el} is the elastic component of *J* integral, J_{pl} is the plastic component of *J* integral. The elastic component of *J* can be directly calculated from the stress intensity factor *K*, as used in ASTM E1820-11 for a plane strain crack

$$J_{el} = \frac{K^2\left(1-\nu^2\right)}{E} \tag{2}$$

in which *E* is the Young's modulus and ν is the Poisson ratio. The plastic component of *J* is determined as

$$\begin{aligned} J_{pl} &= \frac{1}{B}\int_0^P \frac{\partial \Delta_{pl}}{\partial a} dP = -\frac{1}{B}\int_0^\Delta \frac{\partial P}{\partial a} d\Delta_{pl} \\ &= \frac{\eta_\Delta A_\Delta^{pl}}{Bb} \end{aligned} \tag{3}$$

where *P* is the total generalized load or force of the component, *B* is the thickness, *b* is the remaining ligament, *a* is the crack size, Δ_{pl} is a plastic component of load-point or load line displacement, η_Δ is the plastic factor of load line compact tension specimen, A_Δ^{pl} is the plastic component of area under the measured $P-\Delta$ curve.

All expressions introduced above are valid only for stationary cracks. For a growing crack, the *J* integral should consider the crack growth correction. *J* integral is independent of the loading path, so *J* value is a function of two independent variables: Δ and *a* according the deformation theory. From Eq 3, Ernst et al. [18] derived the complete differential of J_{pl} as

$$dJ_{pl} = \frac{\eta_\Delta P}{Bb} d\Delta_{pl} - \frac{\gamma_\Delta}{b} J_{pl} da \tag{4}$$

In which γ_Δ is the geometry factor of load line compact tension specimen. The γ_Δ as follows

$$\gamma_\Delta = \eta_\Delta - 1 - \frac{b}{W}\frac{\eta_\Delta^{'}}{\eta_\Delta} \tag{5}$$

Where the prime denotes the partial differential with respect to *a/W*, i.e. $\eta_\Delta^{'} = \partial \eta_\Delta / \partial\left(a/W\right)$.

Integrating Eq 4, one has

$$J_{pl} = \int_0^{\Delta_{pl}} \frac{\eta_\Delta P}{Bb} d\Delta_{pl} - \int_{a_0}^{a} \frac{\gamma_\Delta}{b} J_{pl} da \tag{6}$$

In which a_0 is the initial crack size.

Figure 1 illustrates a typical $P-\Delta_{pl}$ curve for a growing crack. This figure includes the deformation paths for an original crack length a_0 and also for two arbitrarily fixed crack lengths a_i and a_{i+1}. Since the J_{pl} in Eq 6 is valid for any loading path leading to the current values of a_i and Δ_{pl}^i, its value at point A(or B) can be determined by following path OA (or OB) for the fixed crack length a_i to the corresponding Δ_{pl}^i (or Δ_{pl}^{i+1}) in the actual $P-\Delta$ curve. Because da=0 on this loading path, from Eq 6, one has

$$J_{pl}^A = \frac{\eta_\Delta^i}{Bb_i} \int_0^{\Delta_{pl}^i} P d\Delta_{pl} \tag{7}$$

and

$$J_{pl}^B = J_{pl}^A + \frac{\eta_\Delta^i}{Bb_i} A_{\Delta_{pl}}^{i+1,i} \tag{8}$$

where $A_{\Delta_{pl}}^{i+1,i}$ represents the area under the $P-\Delta_{pl}$ curve between Δ_{pl}^i and Δ_{pl}^{i+1} with an error of the area of triangle ΔABC. Integration of Eq 6 along BC obtains an approximate result

$$J_{pl}^C = J_{pl}^B \left(1 - \frac{\gamma_\Delta^i}{b_i}(a_{i+1} - a_i)\right) \tag{9}$$

From Eqs 7 to 9, one obtains

$$J_{pl(i+1)} = \left(J_{pl(i)} + \frac{\eta_\Delta^i}{Bb_i} A_{\Delta pl}^{i+1,i}\right)\left(1 - \frac{\gamma_\Delta^i}{b_i}(a_{i+1} - a_i)\right) \tag{10}$$

This incremental expression is the LLD-based (Load Line Displacement) *J* estimation equation that was adopted in ASTM E1820-11and all its predecessors, where the specimen thickness *B* is replaced by the net thickness B_N for specimens with side grooves.

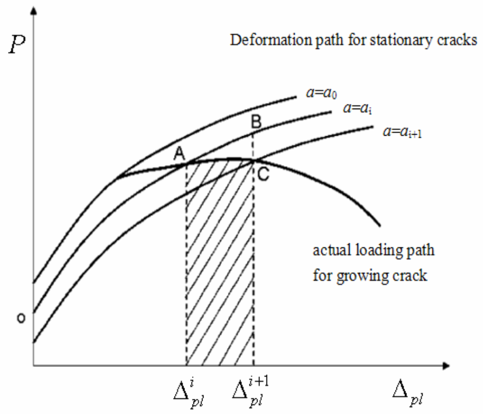

Figure1. Typical load versus plastic displacement curves for static and growing cracks

2.2. Plastic factor of front face compact tension

Following the similar route for deriving the LLD-based *J* equation, this section formulates an incremental CMOD-based (Crack Mouth Open Displacement) *J* estimation. A total CMOD, *V*, is separated into an elastic component V_{el} and a plastic component V_{pl}. As such, the total *J* can be decomposed into an elastic component J_{el} and a plastic component J_{pl}, as show in Eq 1. The J_{el} is defined in Eq 2, whereas the J_{pl} is determined by

$$J_{pl} = \frac{\eta_V A_V^{pl}}{Bb} = \frac{\eta_V}{Bb}\int_0^{V_{pl}} P dV_{pl} \tag{11}$$

where η_V is the plastic factor of front face compact tension, A_V^{pl} is the area under the measured $P-V_{pl}$ curve. Without loss of generality, it is assumed the ratio of V_{pl} and Δ_{pl} is a function of *a/W* [19]

$$\frac{V_{pl}}{\Delta_{pl}} = \lambda\left(a/W\right) \tag{12}$$

Substitution Eq 12 into Eq 11 gives

$$J_{\mathrm{pl}} = \frac{\eta_V}{Bb}\int_0^{\Delta_{pl}} Pd\left(\lambda\Delta_{\mathrm{pl}}\right) = \frac{\eta_V}{Bb}\lambda\int_0^{\Delta_{pl}} Pd\Delta_{\mathrm{pl}} \tag{13}$$

From Eqs 3, 11, 12 and 13, three equivalent expressions for this new plastic factor are obtained

$$\lambda\left(a/W\right) = \frac{V_{\mathrm{pl}}}{\Delta_{\mathrm{pl}}} = \frac{\eta_\Delta}{\eta_V} = \frac{A_V^{pl}}{A_\Delta^{pl}} \tag{14}$$

Since J_{pl} is now the function of V_{pl} and *a*, its complete differential is obtained as

$$dJ_{pl} = \frac{\eta_V P}{Bb} dV_{pl} - \frac{\gamma_V}{b} J_{pl} da \tag{15}$$

where γ_V is the geometry factor of front face compact tension specimen. The γ_V as follows

$$\gamma_V = \lambda \eta_\Delta - 1 - \frac{b}{W}\left(\frac{\lambda'}{\lambda} + \frac{\eta_V'}{\eta_V}\right) \tag{16}$$

Substituting Eq 14 into Eq 16, it is interesting to find that $\gamma_V = \gamma_\Delta$.

Integrating Eq 15 gives the plastic component of *J* in reference to the CMOD-based plastic factor and geometry factor

$$J_{pl} = \int_0^{V_{pl}} \frac{\eta_V P}{Bb} dV_{pl} - \int_{a_0}^{a} \frac{\gamma_V}{b} J_{pl} da \tag{17}$$

As similar to the load line compact tension specimen, we obtain the following CMOD-based *J* estimation equation for a growing crack

$$J_{pl(i+1)} = \left(J_{pl(i)} + \frac{\eta_V^i}{Bb_i} A_{Vpl}^{i+1,i}\right)\left(1 - \frac{\gamma_V^i}{b_i}\left(a_{i+1} - a_i\right)\right) \tag{18}$$

The CMOD-based *J* formulation is the other way to determine *J* resistance curves for 1/2FFCT specimens.

Based on the detailed FEA, the plastic factor of front face compact tension specimen is investigated. The function, λ (*a/W*) , is obtained by linear fitting of the least squares method, considering the difference strain hardening of material mechanical behavior. The λ (*a/W*) is expressed as (be shown as in Fig. 2)

$$\lambda(a/W) = -0.3412\left(a/W\right) + 1.5356 \tag{19}$$

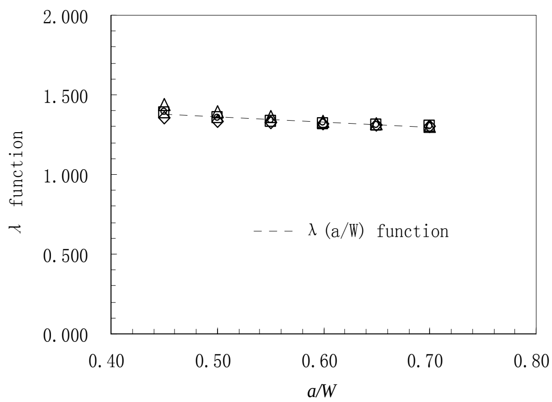

Figure 2. The function λ versus a/W curve

From Eqs 12, 14, the plastic factor of front face compact tension specimen is fitted by the following curve

$$\begin{aligned}\eta_V &= \eta_\Delta / \lambda(a/W) \\ &= 1.6729 - 0.0329\left(1 - a/W\right)\end{aligned} \tag{20}$$

3 Conclusions

1> Based on the FEA, the function λ (*a/W*) is got.

2> Connecting FEA and function λ (*a/W*), the plastic factor formulation of front face compact tension is obtained.

3> Using the plastic factor of front face compact tension specimen, the *J* resistance curve would be perfectly solved by the normalization data reduction technology in ASTM E1820-11.

Based on the plastic hinge theory of bending specimen and the plastic factor of load line compact tension, the unified plastic factor calculation is investigated in future. From the unified plastic factor, considering the normalization data reduction in ASTM E1820-11, the *J* resistance curve would be perfectly solved.

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