Buckling Analysis of a Nanowire Lying on Winkler-Pasternak Elastic Foundation

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Abstract The Steigmann-ogden surface elasticity model and the Timoshenko beam theory are adopted to study the axial buckling of a nanowire (NW) lying on Winkler-Pasternak substrate medium. Explicit solutions of the critical buckling force and buckling mode are obtained analytically. The influences of the surface stress effect, the geometry of the NW and the elastic foundation moduli are systematically discussed.

Keywords Surface effect, Nanowire, Timoshenko beam theory, Buckling

1. Introduction

Nanowire (NW) based devices have found important applications in various fields. Traditional beams theories failed to interpret the size dependent mechanical behavior of these devices due to their surface stress effect. The Gurtin-Murdoch model [1-2] has been widely accepted to study the mechanical behavior of nanostructures and nano defects [3-7]. Some researchers found that the effective elastic moduli of nanostructures under bending and tension are different[8-9], which could not be explained by the Gurtin-Murdoch model. Chhapadia et al.[10] pointed out that the above discrepancy could be explained with the modified framework proposed by Steigmann and Ogden [11-12]. On the other hand, buckling has long been thought as an unwanted issue and should be strictly avoided in structural designs. However, it was demonstrated by some researchers that controlled buckling of slender structures such as thin films, nanowires and nanotubes on compliant substrates could be utilized in the design of flexible electronics. This paper aims to study the axial buckling of a simply supported NW lying on Winkler-Pasternak elastic foundation with the Timoshenko beam model and Steigmann–Ogden theory. The influences of the surface stress effect, the geometry of the NW and the elastic foundation moduli are systematically discussed.

2. Solution of the Problem

Consider a NW lying on a deformable substrate, which is subjected to distributed transverse load and axial forces at both ends. We assume the NW has a circular cross section. The total energy of the axially loaded nanobeam can be written as:

$$U_{Total}(w) = U_{Bulk}(w) + U_{Surface}(w)\,, \tag{1}$$

where:

$$U_{Bulk}(w) = \int_0^L \int_A \frac{1}{2} E\left(y\frac{\partial \psi}{\partial x}\right)^2 dAdx + \int_0^L \int_A \frac{\kappa}{2} G\left(\frac{\partial w}{\partial x} - \psi\right)^2 dAdx \quad , \tag{2}$$

$$U_{Surface}(w) = \int_0^L \int_S \left[-\tau_0 y\frac{\partial \psi}{\partial x} + \frac{1}{2}C_0\left(y\frac{\partial \psi}{\partial x}\right)^2 + \frac{1}{2}C_1\left(\frac{\partial^2 w}{\partial x^2}\right)^2\right] dSdx \quad . \tag{3}$$

Here, A denotes the area of the cross section, κ is the shear coefficient, C_0 and C_1 are the surface elastic modulus and the Steigman-Ogden constant, respectively, S denotes the circumference of the cross section. The potential energy of the lateral and axial load is given by:

$$U_f(w) = -\int_0^L f(x)w(x)dx - \frac{1}{2}\int_0^L N(\frac{\partial w}{\partial x})^2 dx, \tag{4}$$

where

$$f(x) = q(x) + q^s(x) + q^f(x). \tag{5}$$

Here, $q^s(x)$ stands for the equivalent lateral force due to the residual surface stress and $q^f(x)$ represents the distributed force arising from the substrate medium. To minimize the total potential energy, we apply the variational theory,

$$\delta\left[U_{Total}(w) + U_f(w)\right] = 0. \tag{6}$$

Finally, we get:

$$\begin{cases} GA\kappa(\frac{\partial^2 w}{\partial x^2} - \frac{\partial \psi}{\partial x}) + f(x) - N\frac{\partial^2 w}{\partial x^2} = 0 \\ GA\kappa(\frac{\partial w}{\partial x} - \psi) + \frac{\partial}{\partial x}((EI + C_0 I* + C_1 S)\frac{\partial \psi}{\partial x}) = 0 \end{cases}, \tag{7}$$

where $I = \int_A y^2 dA$, $I* = \int_S y^2 dS$, $S* = \int_S n_y^2 dS$. Take partial derivative of the second equation with respect to x, and substitute the first equation into the second equation, we have:

$$\left[(EI)* + \frac{(EI)*}{GA\kappa}(H-N) + \frac{(EI)*}{GA\kappa}C_p\right]\frac{\partial^4 w}{\partial x^4} + \left[N - \frac{(EI)*}{GA\kappa}C_w - H - C_p\right]\frac{\partial^2 w}{\partial x^2} + C_w w(x) = 0. \tag{8}$$

The general solution of Eq. (8) can be written as:

$$w(x) = Ae^{mx}. \tag{9}$$

Substituting Eq. (9) into Eq. (8), we can solve m.

The boundary conditions of a simply supported NW are given as follows:

$$w(0) = 0, \quad w''(0) = 0, \tag{10a}$$

$$w(L) = 0, \quad w''(L) = 0. \tag{10b}$$

Substitute Eqs.(10a,b) into Eq. (9), we can get the characteristic equation to solve the critical buckling force. The corresponding buckling mode is given by Eq. (9).

3. Results and Discussions

The buckling force of the NW is normalized with the critical buckling force of a classic simply supported Euler-Bernoulli beam. To see qualitatively how the surface stress effect and the Steigmann-Ogden correction influence the buckling behavior of the NW, we adopt the following material parameters in this study:

$$E = 76\,\text{Gpa}\,, \upsilon = 0.3\,,\ \ \tau_0 = 0.65\ \text{N/m}\,,\ \ C_0 = -1.39\,\text{N/m}\,,\ \ C_1 = \pm 153.6\,\text{eV}\,.$$

In Figure 1., we ignore the transverse shear effect of the substrate. The diameter and Steigmann-Ogden constant are chosen to be $3.5\,\text{nm}$ and $153.6\,\text{eV}$ respectively. All the results are normalized with the critical buckling force of a classic simply supported Euler-Bernoulli beam at each aspect ratio. Figure 1. shows that the surface effect has significant influences on the normalized critical buckling forces. When the Steigmann-Ogden constant is positive, the critical buckling force is greater than that predicted by the Gurtin-Murdoch theory, which implies that the NW is stiffened. It is also noted that the influence of the surface effect (predicted either by the Gurtin-Murdoch model or the Steigmann-Ogden model) on the critical buckling force decreases when the diameter of the NW increases. The difference between the result predicted by the Gurtin-Murdoch theory and that by the Steigmann-Ogden theory is also largely decreased as the diameter of the NW increases. Fig. 1 also shows that the critical buckling force for a finitely long NW is always larger than that for an infinitely long NW.

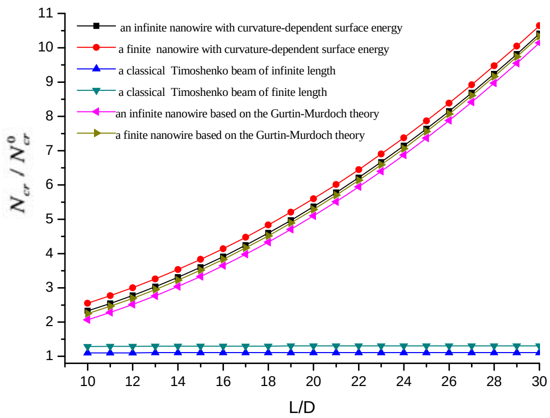

Figure 1. The normalized critical buckling force of a simply support NW lying on Winkler substrate medium.

In Figure 2., the influences of the Winkler and Pasternak moduli on the critical buckling force are studied with the Timoshenko beam model. The material parameters are the same with those in Figure 1.. The diameter of the NW is set to $3.5\,\text{nm}$. From Figure 2., we find that both the Winkler modulus and the Pasternak modulus tend to stiffen the NW. As a result, the buckling force for a NW lying on substrate medium is always larger than that of a free NW. It is also noticed that the critical buckling force is more sensitive to the Pasternak modulus.

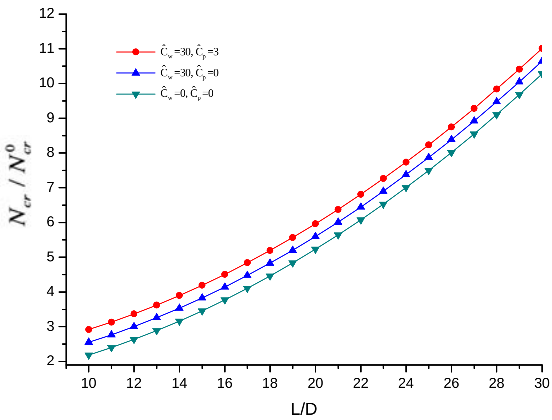

Figure 2. Variation of the normalized critical buckling force of a simply supported NW lying on Winkler-Pasternak substrate medium with respect to its aspect ratio. The diameter of the NW is $3.5\,\text{nm}$.

4. Conclusions

In this paper, the Steigmann-Ogden model is adopted to characterize the surface effect of the NW. Explicit solutions are obtained for the critical buckling force and buckling mode of a simply supported NW lying on Winkler-Pasternak substrate medium with the Timoshenko beam theory. The following conclusions are drawn through this study:

(1) The shear effect, the surface stress effect and curvature dependent surface energy all have influences on the critical buckling force of the NW. The importance of these influences is highly dependent on the diameter and aspect ratio of the NW.

(2) The Steigmann-Ogden correction can stiffen or soften the NW, depending on the sign of the Steigmann-Ogden constant.

(3) Both the Winkler modulus and Pasternak modulus tend to stiffen the NW. The critical buckling force of the NW is more sensitive to the Pasternak modulus.

Acknowledgement

The support of National Natural Science Foundation of China under contract No. 10802032 is most sincerely appreciated.

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