TIME-DERIVATIVE EQUATIONS FOR FATIGUE CRACK GROWTH.

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ABSTRACT

Predicting fatigue crack growth in metals remains a difficult task since available models are based on cycle-derivative equations, such as the Paris law, while service loads are often far from being cyclic. The main objective of this paper is therefore to propose a set of time-derivative equations for fatigue crack growth. The model is based on the thermodynamics of dissipative processes. For this purpose, three global state variables are introduced in order to characterize the state of the crack: the crack length a, the plastic blunting at crack tip and the intensity of crack opening C. Thermodynamics counterparts are introduced for each variable. Special attention is paid to the elastic energy stored inside the crack tip plastic zone, since, in practice, residual stresses at crack tip are known to considerably influence fatigue crack growth. The stored energy is included in the energy balance equation, and this leads to the appearance of a kinematics hardening term in the yield criterion for the cracked structure. No dissipation is associated with crack opening, but to crack growth and to crack tip blunting. Finally, the model consists in two laws: a crack propagation law, which is a relationship between dρ/dt and da/dt and which observes the inequality stemmed from the second principle, and an elastic-plastic constitutive behaviour for the cracked structure, which provides dρ/dt versus applied-load. The model was implemented and tested. It reproduces successfully the main features of fatigue crack growth as reported in the literature, such as the Paris law, the stress ratio effect and the overload retardation effect.

1 INTRODUCTION

Engineering models struggle with loads-load interaction effects in fatigue crack growth. Monotonic fatigue crack growth rates are usually successfully predicted using the Paris law. Under non-monotonic fatigue, damage accumulation rules are employed.

One key problem encountered in variable amplitude fatigue is that, because the Paris law is a cycle-derivative equation, cycles have to be extracted from real signals [1]. However, because of load history effects in fatigue crack growth [2], this operation is questionable. Therefore, a set of time-derivative equations was build for fatigue crack growth modeling under variable amplitude fatigue and taking into account load history effects.

2 MODEL

The model was developed on the basis of the thermodynamics of dissipative processes. Three global variables are employed to characterize the velocity field at crack tip, the crack growth rate da/dt, the rate of plastic blunting dρ/dt and the rate of elastic crack opening. Thermodynamics counterparts are introduced for each velocity variable. Since crack tip residual stresses are well known to considerably influence fatigue crack growth under variable amplitude loading, [2-4], the elastic energy stored inside the crack tip plastic zone is included in the energy balance equation. This equation leads to establish a yield criterion for the cracked structure. In this criterion, the energy stored within the crack tip plastic zone (residual stresses) leads to the appearance of a kinematics hardening term for the cracked structure. Besides, the second principle of thermodynamics provides an inequality between da/dt and dρ/dt, that the instantaneous cracking law employed in the model should observe.

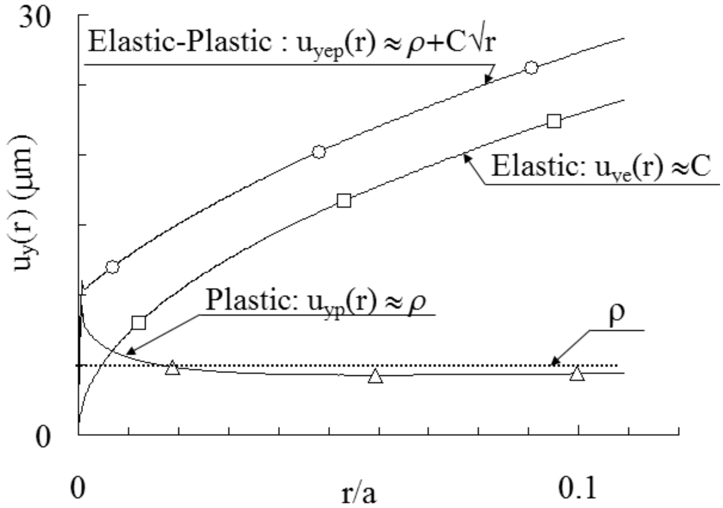


Figure.1. Displacement of the crack faces, as calculated using the finite element method for a stationary crack. 2D. Plane strain. CCT specimen.

Finally the constitutive equations, governing the evolution equations of dρ/dt, are derived using the finite element analyses as an unzooming technique.

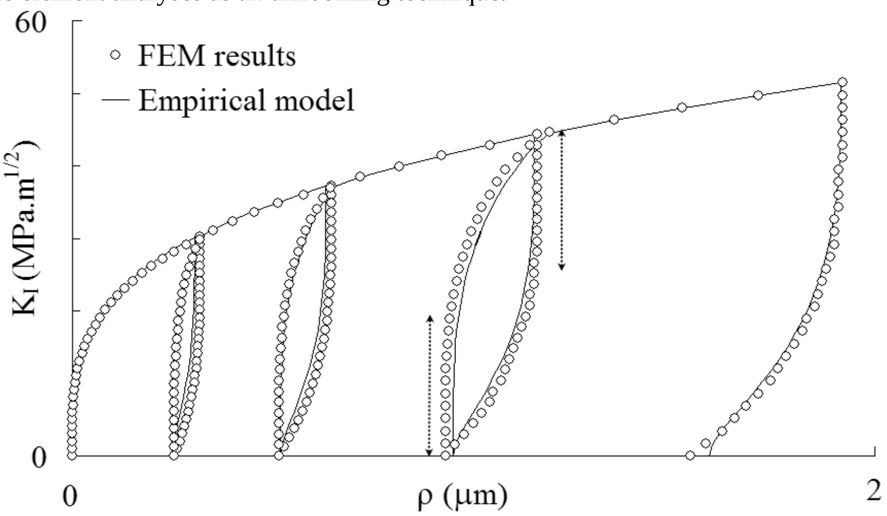


Figure.2. Evolution of crack tip blunting as calculated using the finite element method, versus the applied stress intensity factor.

The crack tip blunting ρ is calculated as the average value of the permanent displacement of the crack faces within the K-dominance area (Fig. 1). Provided that the plastic zone is fully constrained inside the K-dominance area, the difference between the displacement as calculated in elasticity $u_{ye}(r)$ and in elasticity-plasticity $u_{yep}(r)$ yields the permanent displacement $u_{yp}(r)$ of the crack faces after unloading elastically the cracked structure. It appears, in Fig. 1 for instance, that this permanent displacement is almost a constant. The calculation of ρ and of its associated error is thus as follows:

$$\rho = \frac{20}{a}\int_{r=a/20}^{r=a/10}\left(u_{yep}(r)-u_{ye}(r)\right)dr \qquad error = \sqrt{\frac{10}{a}\int_{r=0}^{r=a/10}\left[\left(u_{yep}(r)-u_{ye}(r)\right)-\rho\right]^2 dr}$$

At this stage, the evolution of ρ can be plotted as a function of the applied stress intensity factor (Fig. 2). An elastic domain can be identified for the cracked structure. It varies in size and in location. Empirical equations have been adjusted to the calculated evolutions of crack tip blunting in order to establish an elastic-plastic constitutive behaviour for the cracked structure.

3 RESULTS

The model was implemented and tested. In particular, the evolution of crack tip blunting as calculated using the model and as calculated using the FEM are compared (Fig. 2). After adjusting the material parameters in the equations, the agreement is satisfactory.

At this stage, for instance, it is possible to study, coupling effects between HCF and LCF (Fig. 3). In this graph is plotted the comparison between the evolution of crack tip blunting as calculated using the model and using the FEM for a vibratory stress superimposed with a static stress. The agreement between the model and the FEM is satisfactory for the amplitude of crack tip blunting $\Delta\rho_{HCF}$ during HCF cycles. Besides, a coupling effect appears between HCF and LCF which consist in a slow ratchetting at crack tip during LCF cycles inherited from CHF cycles. This ratchetting effect does not appear is pure LCF fatigue.

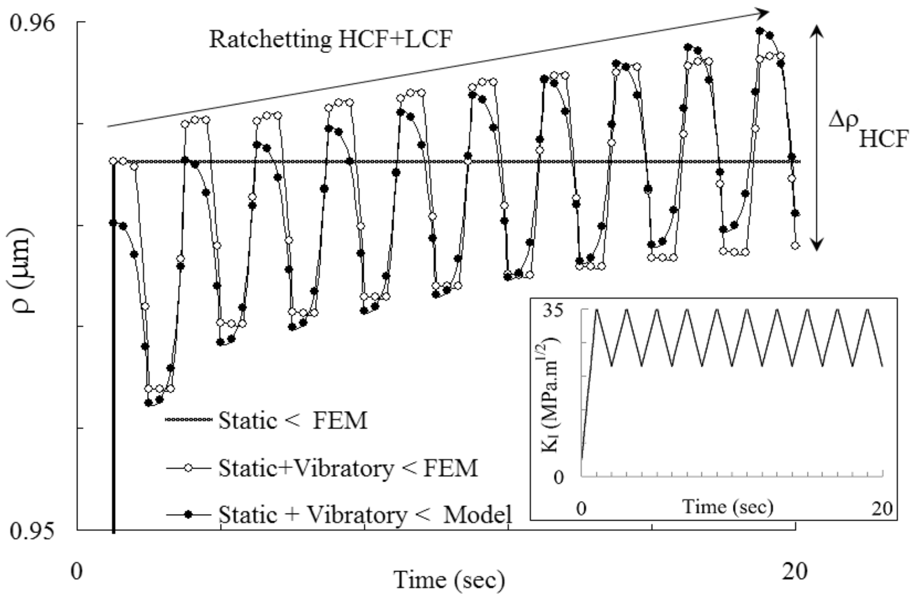


Figure 3: Evolution of crack tip blunting versus time, for a LCF+HCF cycle. In the insert, applied stress intensity factor versus time. Comparison between the FEM and the prediction of the model.

To sum up this section in a few words, the finite element method was employed in order to establish an elastic-plastic constitutive behaviour for the cracked structure. All the material parameters of this model are determined using the finite element method, from the knowledge of the constitutive behaviour of the material only, independently of any fatigue crack growth experiments.

Now, in order to predict also the fatigue crack extent, a relationship between the rate of crack tip blunting and the rate of crack extent was proposed. This relationship observes the inequality stemmed from the second principle of thermodynamics. Also, it can be considered that it derives directly from the ΔCTOD equation, which was proved to be valid by various authors through fracture surface analyses [5-7].

The propagation equation is as follows:

$$\frac{da}{dt} = \frac{\alpha}{2}\left\langle \frac{d\rho}{dt} \right\rangle \quad \textit{with} \quad \alpha \geq 1 \quad \textit{and} \quad \begin{cases} x \geq 0 & \langle x \rangle = x \\ x < 0 & \langle x \rangle = 0 \end{cases}$$

If α is set to one, and if a full constant amplitude cycle is considered, this equation implies that the crack growth rate is of one striation per cycle.

By gathering the equations governing the evolution of crack tip blunting and the propagation equation, it is now possible to calculate the fatigue crack growth versus time. Since the model was build without considering any information from fatigue crack growth experiments, it is necessary to check first of all, if it is consistent with fatigue crack growth results as reported in the literature.

First of all, a constant amplitude test was simulated. The crack growth rate as calculated using the model obeys the Paris law (Fig. 4). In the present case the Paris exponent was found to be m=5. This is related to the value of α which can be adjusted on one experiment. Besides, a sensitivity to the stress ratio is found, which is qualitatively in agreement the literature (Fig. 5). The "crack opening level" is calculated from the Paris laws as obtained with the model, assuming that at R=0.5 , K_{op}/K_{max}=R:

$$\frac{da}{dN} = C_o \Delta K_{eff}^{m} = C_o \left(1 - \frac{K_{op}}{K_{\max}}\right)^m \left(\frac{\Delta K}{1-R}\right)^m = C.\Delta K^{m}$$

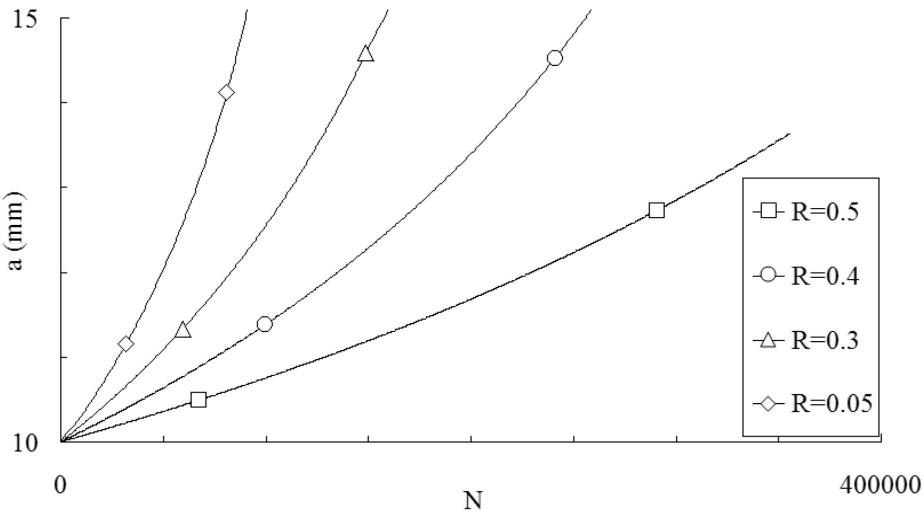
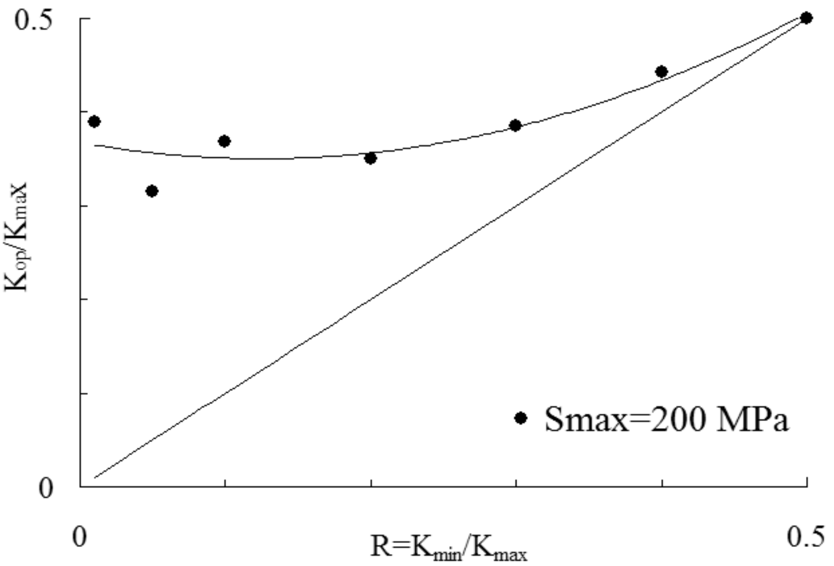


Figure 4: Constant amplitude fatigue. Effect of the Stress ratio R. Evolution of crack length as calculated using the model for various stress ratios, for ao=10 mm, and Smax=200 MPa.



(b)

Figure 5: Constant amplitude fatigue. Effect of the Stress ratio R. Evolution of the “crack opening level”, as calculated using the model for various stress ratios, for ao=10 mm, and Smax=200 MPa. Kop/Kmax is calculated from the crack growth rates as calculated from the results in Fig. 4.

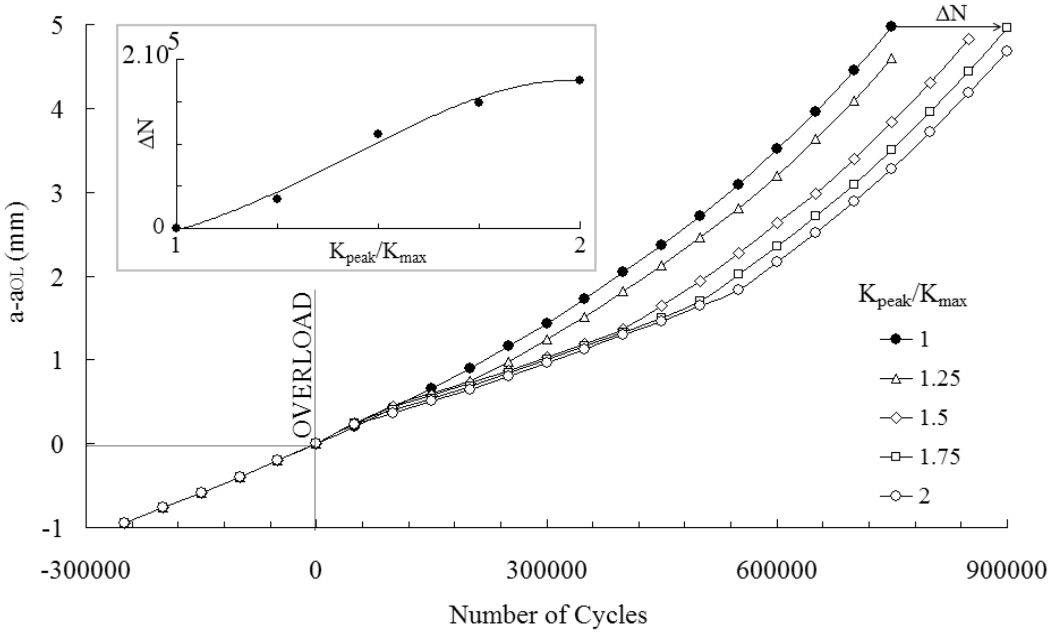


Figure 6: Evolution of the crack length versus the number of cycle, as predicted using the model, after an overload with an overload factor K_{peak}/K_{max} equal to 1, 1.25, 1.5, 1.75 and 2.

Finally the ability of the model to reproduce the overload effect for instance [8] was also tested (Fig. 6). The evolution of the crack length a from the application of an overload a_{OL} is plotted as a function of the number of cycles for various overload ratios. It is found that the overload effect increases with the overload ratio. Besides, it was shown also that the retardation effect diminishes for the same overload ration when the stress intensity factor increases.

All these results, though calculated from a model which was established independently of any fatigue crack growth experiments are qualitatively in agreement with experimental observations.

3 CONCLUSIONS

The present model was developed on the basis of the thermodynamics of dissipative processes. It consist in an elastic-plastic constitutive behaviour for the cracked structure with provides the rate of crack tip blunting versus loads. This constitutive behaviour is established using the FE method, independently of any fatigue crack growth experiments. Besides a propagation equation is introduced which is a relationship between the fatigue crack growth rate and the blunting rate. This equation is derived from the ΔCTOD equation.

The model was integrated and tested. Though the model was defined independently from any crack propagation experiments it allows reproducing successfully the main features of fatigue crack growth, namely the fact that the crack growth rate obeys the Paris law in constant amplitude fatigue, the stress ratio effect, the overload retardation effect and the sensitivity of this retardation effect to the overload ratio and to Kmax.

4 REFERENCES

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