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DYNAMIC FRACTURE AND FRAGMENTATION OF BRITTLE SPHERES SUBJECT TO DOUBLE IMPACT TEST

K.T. Chau , S.Z.Wu and X.X. Wei

Department of Civil and Structural Engineering, The Hong Kong Polytechnic University, Kowloon, Hong Kong, China

ABSTRACT

This paper presents a new model for dynamic fracturing of a brittle sphere subject to the "double impact test", in which the sphere is crushed dynamically between two flat rigid platens. The dynamic tensile stress field in the specimen is calculated analytically using a superposition approach by Valanis [1]. The calculated local tensile field is imposed on vertical microcracks. The crack growth velocity is established in terms of the static stress intensity factor, the dynamic fracture toughness, and the Rayleigh wave speed. The dynamic strength corresponds to the crack growth velocity becoming unbounded. Double impact tests done on plaster spheres were used to verify the present model. Experimental results show that the impact energy required for fracturing in double impact test is about 150% of that required by the static counterpart (i.e. the diametral compression test).

KEYWORDS

Dynamic crack growth, Impact loads, Compression and fragmentation, Spheres

INTRODUCTION

The dynamic fragmentation of brittle spheres under impact loads has a wide range of engineering applications. It is also one of the most fundamental problems in applied mechanics. It relates to phenomena covering a wide range of length scales, from a large length scale of the collisional evolution of asteriods to shorter length scale of the degradation of materials in process industries, such as pharmaceuticals, chemical, fertilizers and detergents. In civil engineering and rock mechanics, its application has been in the mining industry, involving grinding, crushing, and impact comminution (Chau et al. [2]). Fragmentation of boulders during the impact phase of rockfall can also be modeled by dynamic impact of spheres (Chau et al. [3-5]).

Extensive experimental and theoretical efforts have been made to estimate the required energy and force to fracture a brittle solid into fragments with desirable size distribution. In the experimental approach, one of the most popular tests is the compression test of spheres, either statically or dynamically, between two flat rigid platens. Under quasi-static loads, compression of spheres between two flat platens has been proposed for testing the deformability of elastic materials, hardness of ductile materials and crushing strength of brittle materials. For example, the crushing of spheres between the flat platens can also be used to estimate the tensile strength of brittle spheres. A comprehensive review is given by Chau et al. [2] and by Darvell [6]. Although there are numerous experimental studies, stress distribution within a sphere under compression between two rigid platens has not been studied comprehensively. The most popular theoretical model is that proposed by Hiramatsu and Oka [7], which has been applied by various authors. Chau et al. [2] also provided a extension of Hiramatsu-Oka solution to incorporating the Hertz contact stress under compression. For the dynamic impact of spheres between two rigid platens, although a informative crater analysis was also suggested by Chau et al. [2]. However, due to mathematical complexity, Chau et al. [2] did not consider the exact solution for the stress distribution within spheres under the double impact test. Therefore, the dynamic stress inside the sphere and its relationship to the final fragmentation is not well understood.

Therefore, this paper outlines a new approach in which the Valanis [1] superposition principle for dynamic problems and the dynamic crack growth results considered by Freund [8] are combined to investigate the problem of dynamic fragmentation in spheres. The analysis is still on-going, only the essential idea and preliminary results will be reported.

VALANIS (1966) SUPERPOSITION PRINCIPLE

By applying the superposition principle put forward by Valanis [1], the problem of double impact test on spheres can be decomposed into two associated problems: the static problem and the free vibration problem of a sphere.

Static Compression of Spheres

The static solution for sphere can be generalized from that of Hiramatsu and Oka [7] (see Chau et al. [2]). In particular, by incorporating the Hertz contact stress, the static problem of compression of sphere between two rigid platens are:

$$s_{rr} = \sum_{n=0}^{\infty} P_{2n}(\cos \mathbf{q}) \left\{ -\frac{\left(4n^2-2n-3\right)\mathbf{l} + 2(2n+1)(n-1)\mathbf{m}}{4n+3} A_{2n} r^{2n} + 4n(2n-1)\mathbf{m} C_{2n} r^{2n-2} \right\} \tag{1}$$

$$\begin{aligned} s_{qq} &= \sum_{n=0}^{\infty} P_{2n}(\cos \mathbf{q}) \left\{ \frac{(2n+3)\mathbf{l} - 2(n-1)\mathbf{m}}{4n+3} A_{2n} r^{2n} + 4n\mathbf{m} C_{2n} r^{2n-2} \right\} \\ &\quad + 2\frac{\partial^2}{\partial \mathbf{q}^2} P_{2n}(\cos \mathbf{q}) \left\{ -\frac{(2n+3)\mathbf{l} + (2n+5)\mathbf{m}}{2(2n+1)(4n+3)} A_{2n} r^{2n} + \mathbf{m} C_{2n} r^{2n-2} \right\} \end{aligned} \tag{2}$$

$$s_{rq} = \sum_{n=0}^{\infty} \frac{\partial}{\partial \mathbf{q}} P_{2n}(\cos \mathbf{q}) \left\{ -\frac{4n(n+1)\mathbf{l} + \left(4n^2+4n-1\right)\mathbf{m}}{(2n+1)(4n+3)} A_{2n} r^{2n} + 2(2n-1)\mathbf{m} C_{2n} r^{2n-2} \right\} \tag{3}$$

where the unknown constants can be obtained from the boundary condition as

$$\begin{cases} A_{2n} = \dfrac{E_{2n}}{a^{2n}} \left\{ \dfrac{(4n+3)(2n+1)}{(8n^2+8n+3)\lambda + 2(4n^2+2n+1)\mu} \right\} & A_{2n+1} = 0 \\ C_{2n} = \dfrac{E_{2n}}{a^{2n-2}} \left\{ \dfrac{4n(n+1)\lambda + (4n^2+4n-1)\mu}{2\mu(2n-1)[(8n^2+8n+3)\lambda + 2(4n^2+2n+1)\mu]} \right\} & C_{2n+1} = 0 \\ E_{2n} = -\dfrac{3(4n+1)F\sqrt{a^2-a_0^2}}{2\pi a_0^3} \displaystyle\int_{\cos\theta_0}^{1} \left(\dfrac{x^2}{\cos^2\theta_0} - 1 \right)^{1/2} P_{2n}(x)dx & \\ \cos\theta_0 = \dfrac{\sqrt{a^2-a_0^2}}{a} \qquad a_0 = \left(\dfrac{3Fa(1-\nu^2)}{4E} \right)^{1/3} \qquad a_0 = a\sin\theta_0 & \end{cases} \tag{4}$$

Free Vibration Problems of Spheres

Instead of using wave potential approach (Chau [11]), the Helmholtz decomposition theorem will be used here to solve the following "reduced dynamics problem" (Achenbach [12]):

$$(\lambda + 2\mu)\nabla\nabla\cdot\vec{V} - \mu\nabla\times\nabla\times\vec{V} = \rho\frac{\partial^2\vec{V}}{\partial t^2} \tag{5}$$

$$\sigma_{rr} = 0 \qquad \sigma_{r\theta} = 0 \quad on \quad r = a; \qquad V_i = U_i, \quad \frac{\partial V_i}{\partial t} = 0 \quad at \quad t=0$$

In particular, we can assume the displacement vector as

$$\vec{V} = \nabla\varphi + \nabla\times\vec{\psi} \tag{6}$$

These scalar and vector potentials satisfy

$$\nabla^2\varphi = \frac{1}{c_1^2}\frac{\partial^2\varphi}{\partial t^2} \qquad \nabla^2\vec{\psi} = \frac{1}{c_2^2}\frac{\partial^2\vec{\psi}}{\partial t^2} \tag{7}$$

For our axisymmetric problem, the second of these becomes

$$\nabla^2\psi - \frac{\psi}{r^2\sin^2\theta} = \frac{1}{c_2^2}\frac{\partial^2\psi}{\partial t^2} \qquad (i.e.\, \vec{\psi} = (0, 0, \psi)) \tag{8}$$

The general solutions for and are:

$$\varphi = \sum_{n=1}^{\infty} A_n r^{-\frac{1}{2}} J_{n+\frac{1}{2}}\left(\frac{k}{c_1} r\right) P_n(\cos\theta) f_n(t), \quad \psi = \sum_{n=1}^{\infty} B_n r^{-\frac{1}{2}} J_{n+\frac{1}{2}}\left(\frac{k}{c_2} r\right) P_n^1(\cos\theta) f_n(t) \tag{9}$$

Substitution of these solutions to the displacement-strain and stress-strain relations leads to

$$\sigma_{rr} = \sum_{n=1}^{\infty} 2\mu A_n F_n(r,k) P_n(x) f_n(t) \tag{10}$$

$$F_n(r,k) = \left\{ \left[n(n-1) - \frac{1-\nu}{1-2\nu}\left(\frac{k}{c_1}r\right)^2 \right] J_{n+\frac{1}{2}}\left(\frac{k}{c_1}r\right) + 2\frac{k}{c_1}rJ_{n+\frac{3}{2}}\left(\frac{k}{c_1}r\right) \right\} r^{-\frac{5}{2}} + \frac{B_n}{A_n}n(n+1)\left[(n-1)J_{n+\frac{1}{2}}\left(\frac{k}{c_2}r\right) - \frac{k}{c_2}rJ_{n+\frac{3}{2}}\left(\frac{k}{c_2}r\right)\right] r^{-\frac{5}{2}} \tag{11}$$

$$\sigma_{r\theta} = \sum_{n=1}^{\infty} \mu A_n G_n(r,k) P_n^1(x) f_n(t) \tag{12}$$

$$G_n(r,k) = -2\left[(n-1)J_{n+\frac{1}{2}}\left(\frac{k}{c_1}r\right) - \frac{k}{c_1}rJ_{n+\frac{3}{2}}\left(\frac{k}{c_1}r\right)\right] r^{-\frac{5}{2}} + \frac{B_n}{A_n}\left[\left(-2n^2+2+\frac{k^2}{c_2^2}r^2\right)J_{n+\frac{1}{2}}\left(\frac{k}{c_2}r\right) - 2\frac{k}{c_2}rJ_{n+\frac{3}{2}}\left(\frac{k}{c_2}r\right)\right] r^{-\frac{5}{2}} \tag{13}$$

$$\sigma_{\theta\theta} = \sum_{n=1}^{\infty} 2\mu A_n \left(H_{n1}P_n(\cos\theta) + H_{n2}\frac{\partial^2}{\partial\theta^2}P_n(\cos\theta)\right) f_n(t) \tag{14}$$

$$H_{n1} = \left\{ \left[\frac{\nu}{-1+2\nu}\left(\frac{k}{c_1}r\right)^2 + n \right] J_{n+\frac{1}{2}}\left(\frac{k}{c_1}r\right) - \left(\frac{k}{c_1}r\right)J_{n+\frac{3}{2}}\left(\frac{k}{c_1}r\right) + \frac{B_n}{A_n}n(n+1)J_{n+\frac{1}{2}}\left(\frac{k}{c_2}r\right) \right\} r^{-\frac{5}{2}}$$

$$H_{n2} = \left\{ J_{n+\frac{1}{2}}\left(\frac{k}{c_1}r\right) + \frac{B_n}{A_n}\left[(n+1)J_{n+\frac{1}{2}}\left(\frac{k}{c_2}r\right) - \left(\frac{k}{c_2}r\right)J_{n+\frac{3}{2}}\left(\frac{k}{c_2}r\right)\right] \right\} r^{-\frac{5}{2}} \tag{15}$$

By applying the traction free conditions on the spherical surface (i.e. $\sigma_{rr} = 0, \quad \sigma_{r\theta} = 0 \quad$ on $r = a$), the following characteristic equations is obtained:

$$2n(n+1)\left[(n-1)J_{n+\frac{1}{2}}\left(\frac{k}{c_2}a\right) - \frac{k}{c_2}aJ_{n+\frac{3}{2}}\left(\frac{k}{c_2}a\right)\right]\left[(n-1)J_{n+\frac{1}{2}}\left(\frac{k}{c_1}a\right) - \frac{k}{c_1}aJ_{n+\frac{3}{2}}\left(\frac{k}{c_1}a\right)\right] + \left\{\left[n(n-1) - \frac{1-\nu}{1-2\nu}\left(\frac{k}{c_1}a\right)^2\right]J_{n+\frac{1}{2}}\left(\frac{k}{c_1}a\right) + 2\frac{k}{c_1}aJ_{n+\frac{3}{2}}\left(\frac{k}{c_1}a\right)\right\} \left[\left(-2n^2+2+\frac{k^2}{c_2^2}a^2\right)J_{n+\frac{1}{2}}\left(\frac{k}{c_2}a\right) - 2\frac{k}{c_2}aJ_{n+\frac{3}{2}}\left(\frac{k}{c_2}a\right)\right] = 0 \qquad (n = 1, 2, 3, \cdots) \tag{16}$$

The full details of analysis and the numerical results will be presented in our forthcoming publication. Nevertheless, a new approach has been outlined here to solve the problem of dynamic compression of sphere.

DYNAMIC CRACK GROWTH IN SPHERE

The solution considered in the previous section for dynamic tensile along the center-line of the sphere can be used to estimate the dynamic mode I stress intensity factor of a vertical microcrack as shown in Fig. 1 (Deng and Nemat-Nasser [13]; Freund [8]):

$$K_{ID} = \sigma_{hoop}(t)\sqrt{\pi a}\left(1 - \frac{\dot{l}}{c_R}\right)\left(1 - \frac{\dot{l}}{2c_R}\right)^{-1} \tag{17}$$

where K_{ID} is the dynamic stress intensity factor, $\sigma_{hoop}(t)$ is the maximum hoop stress along the center-line of the axis of compression obtained in the previous section, $\dot{l}$ is the velocity of crack growth, a is the half size of the microcrack and c_R is the Rayleigh wave speed. It has been assumed that the microcrack is relatively small comparing to the size of the sphere such that local tensile field can be considered as a far field uniform stress. The speed of crack growth can be non-uniform. We should also emphasized that the speed of crack growth should not exceed the Rayleigh wave speed. It can be shown that the speed of crack growth can be determined as (Deng and Nemat-Nasser, [13])

$$\dot{l} = c_R \frac{K_{IS} - K_{IC}}{K_{IS} - \frac{1}{2}K_{IC}} \tag{18}$$

where K_{IS} is the static mode I stress intensity factor and K_{IC} is the dynamic fracture toughness.

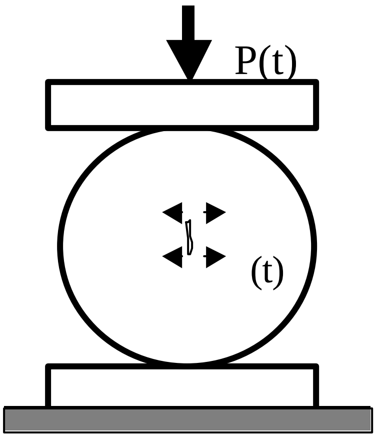

Figure 1: Dynamic growth of vertical microcrack in sphere under double impact test.

Chau et al. (2000) showed that the dynamic energy required for fragmentation can be estimated as 1.5 times of that required for static compression. Figure 2 was extracted from Chau et al. (2000).

FUTURE WORK TO BE DONE

In the case of double impact test, the present approach described needs to be combined with the dynamic motion of the drop weight attached to the upper platen. The application of the contact force during the dynamic impact needs to be evaluated by applying Newton second law to the falling rigid upper platen.

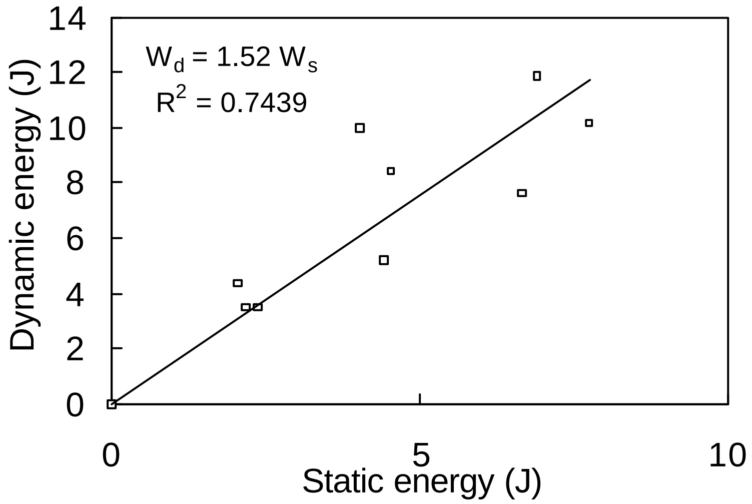

Figure 2: Dynamic energy versus static energy required for fragmentation of sphere (after Chau et al. [2])

CONCLUSION

The present paper summarizes a new theoretical approach to consider the dynamic stress within a sphere under double dynamic test. The application of this tensile stress to dynamic fracture in the sphere is also outlined. More elaborated numerical analysis remains to be done and will be presented at a later time. Nevertheless, the present framework should be very useful to investigate the dynamic fragmentation problem of spheres under dynamic compressions.

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